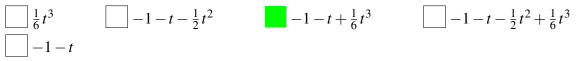
Name:
 Student ID:
 Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. Which of the following ODE's has distinct solutions $y_1, y_2: [-1,1] \to \mathbb{R}$ satisfying $y_1(0) = y_2(0)$? $y' = \sqrt{y^2 + 1} \qquad |t^2y'' = y \qquad |y' = \sqrt{|t|}y \qquad |y'| = t |y|$ 2. The ODE $(y^2 + y) dx - x dy$ has the integrating factor $0 \qquad x^{-1} \qquad y^{-1} \qquad x^{-2}$ y^{-2} 3. For the solution y(t) of the IVP $y' = y^4 - 4y^2$, y(4.29) = 1 the limit $\lim_{t \to +\infty} y(t)$ equals $-\infty$ -2 0 2 $+\infty$ 4. For the solution y(t) of the IVP $y' = y \ln t$, y(1) = 1 the value y(e) is equal to e^{-2} e^{-1} 1 e^{-2} e^2 5. For the solution y: $(0,\infty) \to \mathbb{R}$ of the IVP $t^2 y'' - t y' + y = 1$, y(1) = y'(1) = 0 the value y(e) is equal to -1 $1+\ln 4$ $-1+\ln 4$ $\ln 4$ 1 6. The power series $z + \frac{1}{2}z^2 + \frac{1}{4}z^4 + \frac{1}{8}z^8 + \frac{1}{16}z^{16} + \cdots$ has radius of convergence $0 \qquad 1 \qquad \sqrt{2} \qquad 2$ ∞ 7. The largest integer *s* such that $f_s(x) = \sum_{n=1}^{\infty} \frac{\cos(n^s x)}{n^3}$ is differentiable on \mathbb{R} is equal to 0 1 2 3 44 8. For which choice of $f_n(x)$ does the function series $\sum_{n=1}^{\infty} f_n$ converge uniformly on \mathbb{R} ? $f_n(x) = \sqrt[n]{x^2}/n^4 \qquad f_n(x) = n/(x^4 + n^2) \qquad f_n(x) = \sqrt{x^2 + n}/n^4$ $f_n(x) = n/(x^2 + n^4) \qquad f_n(x) = (-1)^n \ln(x^2 + n)/n^4$ 9. If y(t) solves $y' = \frac{y+2t}{y+t}$ then z(t) = y(t)/t solves $\Box z' = \frac{2-z^2}{z+1}$

Continued on the back side

10. The sequence $\phi_0, \phi_1, \phi_2, \dots$ of Picard-Lindelöf iterates for the IVP y' = y + t, y(0) = -1 has $\phi_2(t)$ equal to



- 11. $y'' + y = \cos t$ has a particular solution $y_p(t)$ of the form ct $c \cos t$ $c \sin t$ $ct \cos t$ $ct \sin t$ with a constant c.
- 12. Maximal solutions of $y' = y^5 + y$ satisfying y(0) = 1 are defined on an interval of the form $[a,b] [a,b] [a,b] [a,+\infty) [a,+\infty$

13. For $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$ and $t \in \mathbb{R}$, the matrix $e^{\mathbf{A}t}$ is equal to $\begin{pmatrix} (1-t)e^t & -te^t \\ te^t & (1+t)e^t \end{pmatrix} \qquad \Box \begin{pmatrix} 1 & e^t \\ e^{-t} & e^{2t} \end{pmatrix} \qquad \Box \begin{pmatrix} (1+t)e^t & te^t \\ -te^t & (1-t)e^t \end{pmatrix}$ $\Box \begin{pmatrix} (1-t)e^t & te^t \\ -te^t & (1+t)e^t \end{pmatrix} \qquad \Box \begin{pmatrix} (1+t)e^t & -te^t \\ te^t & (1-t)e^t \end{pmatrix}$

- 14. The matrix norm of $\begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$ (subordinate to the Euclidean length on \mathbb{R}^2) is contained in the interval [1,2] [2,3] [3,4] [4,5] [5,6]
- 15. A map $T: M \to M$ satisfying $|T(x) T(y)| \le \frac{2021}{2022} |x y|$ for all $x, y \in M$ must have a fixed point if M is equal to

All intervals are in \mathbb{R} , and \mathbb{Q} is the field of rational numbers.

[0,1)

16. This midterm exam was too easy too difficult too long too short just appropriate

Time allowed: 50 min

 $|(0,+\infty)|$

CLOSED BOOK

 $[0, +\infty)$

Good luck!

Q

(0,1)