Math 286 Differential Equations Plus (Prof. Honold) **Midterm Exam** 2022/04/29

Name: ____________________ Student ID: ____________________ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. Which of the following ODE's has distinct solutions $y_1, y_2\colon [-1,1] \to \mathbb{R}$ satisfying $y_1(0) = y_2(0)$?

   ☐ $y' = \sqrt{y^2+1}$ ☐ $t^2y'' = y$ ☐ $y' = \sqrt{|t|}\,y$ ■ $(y')^3 = y$ ☐ $y' = t\,|y|$

2. The ODE $(y^2+y)\,\mathrm{d}x - x\,\mathrm{d}y$ has the integrating factor

   ☐ 0 ☐ x^{-1} ☐ y^{-1} ☐ x^{-2} ■ y^{-2}

3. For the solution $y(t)$ of the IVP $y' = y^4 - 4y^2,\ y(4.29) = 1$ the limit $\lim_{t\to+\infty} y(t)$ equals

   ☐ $-\infty$ ☐ -2 ■ 0 ☐ 2 ☐ $+\infty$

4. For the solution $y(t)$ of the IVP $y' = y\ln t,\ y(1) = 1$ the value $y(\mathrm{e})$ is equal to

   ☐ e^{-2} ☐ e^{-1} ☐ 1 ■ e ☐ e^2

5. For the solution $y\colon (0,\infty) \to \mathbb{R}$ of the IVP $t^2y'' - t\,y' + y = 1,\ y(1) = y'(1) = 0$ the value $y(\mathrm{e})$ is equal to

   ☐ $\ln 4$ ■ 1 ☐ -1 ☐ $1+\ln 4$ ☐ $-1+\ln 4$

6. The power series $z + \frac{1}{2}z^2 + \frac{1}{4}z^4 + \frac{1}{8}z^8 + \frac{1}{16}z^{16} + \cdots$ has radius of convergence

   ☐ 0 ■ 1 ☐ $\sqrt{2}$ ☐ 2 ☐ ∞

7. The largest integer s such that $f_s(x) = \sum_{n=1}^{\infty} \frac{\cos(n^s x)}{n^3}$ is differentiable on $\mathbb{R}$ is equal to

   ☐ 0 ■ 1 ☐ 2 ☐ 3 ☐ 4

8. For which choice of $f_n(x)$ does the function series $\sum_{n=1}^{\infty} f_n$ converge uniformly on $\mathbb{R}$?

   ☐ $f_n(x) = \sqrt[n]{x^2}/n^4$ ☐ $f_n(x) = n/(x^4+n^2)$ ☐ $f_n(x) = \sqrt{x^2+n}/n^4$
   ■ $f_n(x) = n/(x^2+n^4)$ ☐ $f_n(x) = (-1)^n \ln(x^2+n)/n^4$

9. If $y(t)$ solves $y' = \frac{y+2t}{y+t}$ then $z(t) = y(t)/t$ solves

   ■ $z' = \frac{2-z^2}{t(z+1)}$ ☐ $z' = \frac{z^2-2}{t^2(z+1)}$ ☐ $z' = \frac{z+2}{z+1}$ ☐ $z' = \frac{z+2}{t(z+1)}$
   ☐ $z' = \frac{2-z^2}{z+1}$

Continued on the back side

10. The sequence $\phi_0, \phi_1, \phi_2, \ldots$ of Picard-Lindelöf iterates for the IVP $y' = y + t,\ y(0) = -1$ has $\phi_2(t)$ equal to

☐ $\frac{1}{6}t^3$ ☐ $-1-t-\frac{1}{2}t^2$ ■ $-1-t+\frac{1}{6}t^3$ ☐ $-1-t-\frac{1}{2}t^2+\frac{1}{6}t^3$
☐ $-1-t$

11. $y'' + y = \cos t$ has a particular solution $y_p(t)$ of the form

☐ ct ☐ $c\cos t$ ☐ $c\sin t$ ☐ $ct\cos t$ ■ $ct\sin t$
with a constant c.

12. Maximal solutions of $y' = y^5 + y$ satisfying $y(0) = 1$ are defined on an interval of the form

☐ (a,b) ☐ $[a,b]$ ☐ $(a,+\infty)$ ■ $(-\infty,b)$ ☐ $(-\infty,+\infty)$
with $a, b \in \mathbb{R}$.

13. For $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$ and $t \in \mathbb{R}$, the matrix $e^{\mathbf{A}t}$ is equal to

■ $\begin{pmatrix} (1-t)e^t & -t\,e^t \\ t\,e^t & (1+t)e^t \end{pmatrix}$ ☐ $\begin{pmatrix} 1 & e^t \\ e^{-t} & e^{2t} \end{pmatrix}$ ☐ $\begin{pmatrix} (1+t)e^t & t\,e^t \\ -t\,e^t & (1-t)e^t \end{pmatrix}$
☐ $\begin{pmatrix} (1-t)e^t & t\,e^t \\ -t\,e^t & (1+t)e^t \end{pmatrix}$ ☐ $\begin{pmatrix} (1+t)e^t & -t\,e^t \\ t\,e^t & (1-t)e^t \end{pmatrix}$

14. The matrix norm of $\begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$ (subordinate to the Euclidean length on $\mathbb{R}^2$) is contained in the interval

☐ $[1,2]$ ☐ $(2,3]$ ■ $(3,4]$ ☐ $(4,5]$ ☐ $(5,6]$

15. A map $T\colon M \to M$ satisfying $|T(x) - T(y)| \le \frac{2021}{2022}|x-y|$ for all $x, y \in M$ must have a fixed point if M is equal to

☐ $(0,+\infty)$ ☐ $[0,1)$ ■ $[0,+\infty)$ ☐ $(0,1)$ ☐ $\mathbb{Q}$
All intervals are in $\mathbb{R}$, and $\mathbb{Q}$ is the field of rational numbers.

16. This midterm exam was

☐ too easy ☐ too difficult ☐ too long ☐ too short ☐ just appropriate

Time allowed: 50 min CLOSED BOOK ***Good luck!***