Midterm 1 – off campus

Date: November 5, 2021, 19:00-20:00

Name	Student ID

This exam contains eight exercises, each with 5 points. You have to select five of these eight exercises. 25 points are counted as 100%.

EXERCISE 1. Show that for arbitrary sets A, B, C we have $A - (B \cup C) = (A - B) \cap (A - C)$.

total points: 5

EXERCISE 2. Define a binary relation $\preceq$ on $\mathbb{N} \times \mathbb{N}$ by $(i_1, j_1) \preceq (i_2, j_2)$ iff $(i_1 \leq i_2 \wedge j_1 \leq j_2)$. Show the $\preceq$ is a partial order, but not a total order.

total points: 5

EXERCISE 3. Look at the truth table at the right. Find a propositional formula for φ using propositional atoms p, q, r. Then use the Quine-McCluskey method to simplify the formula.

p	q	r	φ
T	**T**	**T**	**T**
T	**T**	**F**	**F**
T	**F**	**T**	**T**
T	**F**	**F**	**F**
F	**T**	**T**	**T**
F	**T**	**F**	**F**
F	**F**	**T**	**T**
F	**F**	**F**	**T**

total points: 5

EXERCISE 4. Formalise the following statement by a formula in predicate logic:

> There exists a chef of a restaurant with three stars who visits other restaurants with at least one star at least once per month.

total points: 5

Exercise 5.

Prove by induction that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ holds for all $n \in \mathbb{N}$.

total points: 5

EXERCISE 6. Consider a set P of sequences of characters that is inductively defined as the smallest set satisfying the following properties:

(i) The empty sequence ε is an element of P;

(ii) Every sequence of length one with a character in the alphabet $A = \{a, e, h, i, k, m, n, o, p, r, t, u, w\}$ is an element of P;

(iii) Whenever a sequence $w \in P$ and a character $x \in A$ are given, then the composed sequence xwx is an element of P.

Show by structural induction that every sequence of characters $w \in P$ is a palindrome, i.e. $w = w^{-1}$, where w^{-1} is the inverted sequence written backwards from the last character in w to the first.

total points: 5

EXERCISE 7. Find all solutions of the following system of linear congruences:

$$x \equiv 4 \mod 5 \qquad x \equiv 2 \mod 8 \qquad x \equiv 2 \mod 3 \; .$$

total points: 5

Exercise 8.

Show $2^{n+1} \in O(2^n)$ and $2^{2n} \notin O(2^n)$.

total points: 5