ECE 313
In-class activity3 Solution

Problem 1

a): According the formula $E(x) = \sum_i a_i * P(x = a_i)$

$$E(U) = \sum_{i=1}^{r} a_i * p_i$$

We know that E(U) = 0, $all\ p_i \geq 0$, $all\ a_i \geq 0$

So, $p_i * a_i \geq 0$ for all i,

If $\exists\, m \neq 0$, $p_m > 0,\ then\ a_m * p_m > 0$, $E(U) > 0$

So, for all i, $i \neq 0$, $p_i = 0, so\ P(U = 0) = 1$.

Proved.

b): According the formula $Var(X) = E[(X - E[X])^2]$

We know that $(V - E[V])^2 \geq 0$, according to the theorem in question (a)

$$P((V - E[V])^2 = 0) = 1$$

So,

$$P(V = E[V]) = 1$$

Proved.

Problem 2

First, find the inverse of the distribution function F(x).

For $x \geq 0$, $F(x) = 1 - \exp(-5x^2)$**,** F(x) = 0, else.

Setting F(x) = u where 0 < u < 1,

$$1 - \exp(-5x^2) = \mathrm{u}$$
$$x = \sqrt{-\frac{1}{5}\ln(1-u)}$$

Therefore, the inverse function $F^{-1}(u) = \sqrt{-\frac{1}{5}\ln(1-u)}$

To construct a random variable X with this distribution from a U(0,1), we can use

$$X = \sqrt{-\frac{1}{5}\ln(1-U)}$$

where U is a uniform distribution U(0,1) and X is the desired random variable.

Problem 3

let Z be the random variable obtained by rounding Y to the nearest integer greater than Y, the distribution of Z is a discrete distribution, and the probability mass function (PMF) can be derived as follows:

The probability that Z= i is equal to the probability that Y falls between i − 1 and i, for i = 1, 2, 3, …. mathematically,

$$P(Z = i) = P(i-1 < Y \le i) = F_Y(i) - F_Y(i-1)$$

The CDF of Y is $F_Y(y) = 1 - e^{-\lambda y}$

So, the distribution of Z is

$$P(Z = i) = e^{-\lambda(i-1)} - e^{-\lambda i}, \text{ i = 1, 2, 3, …….}$$

Problem 4

For Y = -X

$$F_Y(y) = P(Y \le y) = P(-x \le y) = P(X \ge -y) = 1 - F_x(-y)$$

$$f_Y(y) = \frac{d}{dy}F_Y(y) = f_x(-y)$$

For Z = $\frac{1}{X}$, for z > 0

$$F_Z(z) = P(Z \le z) = P\left(\frac{1}{X} \le z\right) = P(X \le 0) + P\left(X > \frac{1}{z}\right)$$

$$= F_X(0) + \left(1 - F_X\left(\frac{1}{z}\right)\right)$$

$$f_Z(z) = \frac{d}{dZ}F_Z(z) = \frac{1}{z^2}f_X(\frac{1}{z})$$

For z < 0

$$F_Z(z) = P(Z \leq z) = P\left(\frac{1}{X} \leq z\right) = P\left(\frac{1}{z} < X < 0\right) = F_X(0) - F_X\left(\frac{1}{z}\right)$$

$$f_Z(z) = \frac{d}{dZ}F_Z(z) = \frac{1}{z^2}f_X(\frac{1}{z})$$