Group Activity 1

Problem 1 :

1) A, E, I, O, U, are vowels
So, $P(\text{first three letters are vowels}) = \dfrac{\binom{5}{3}}{\binom{26}{3}} = \dfrac{1}{260}$

2) There are three possibilities : (1) V, C, V , (2) C, V, V. (3) V, V, C

$P(\text{two vowels and one consonant}) = \frac{5}{26} \times \frac{21}{25} \times \frac{4}{24} + \frac{21}{26} \times \frac{5}{25} \times \frac{4}{24} + \frac{5}{26} \times \frac{4}{25} \times \frac{21}{24}$

$= \frac{21}{260}$

3) Because there are no identical letters in the bag. $P = 0$.

Now consider two alphabet in the bag.

1) A, E, I, O, U, are vowels
So, $P(\text{first three letters are vowels}) = \dfrac{\binom{10}{3}}{\binom{52}{3}} = \dfrac{6}{1105}$

2) There are three possibilities : (1) V, C, V , (2) C, V, V. (3) V, V, C

$P(\text{two vowels and one consonant}) = 3 \times \left(\frac{10}{52}\right) \times \left(\frac{9}{51}\right) \times \left(\frac{42}{50}\right) = \frac{189}{2210}$

3) $P(\text{palindrome}) = \frac{52}{52} \times \frac{50}{51} \times \frac{48}{50} \times \frac{1}{49} \times \frac{1}{48} = \frac{1}{51 \times 49}$

Group Activity 2

We define the event T0 = "a 0 is transmitted", and event R0 = "a 0 is received." Then let T1 = "a 1 is transmitted", and event T1 = "a 1 is received.

Then the events of interest in parts a), b) and c), respectively, are R0, [T1 | R1], [T1 | R0].

An error in the transmitted signal is the union of two disjoint events [T1 $\cap$ R0] and [T0 $\cap$ R1].

From the problem, we have: P(R0 | T0) = 0.9, and P(R1 | T1) = 0.85, and P(T0) = 0.45. From these we get:

$$P(R1 \mid T0) = P(R\overline{0} \mid T0) = 1 - P(R0 \mid T0) = 0.1$$
$$P(R0 \mid T1) = P(R\overline{1} \mid T1) = 1 - P(R1 \mid T1) = 0.15$$
$$P(T1) \quad = 1 - P(T0) = 0.55$$

Now from the theorem of total probability:

$$P(R0) = P(R0 \mid T0).\ P(T0) + P(R0 \mid T1).\ P(T1)$$
$$= (0.9) \times (0.45) + (0.15) \times (0.55) = 0.4875$$

$$P(R1) = P(R1 \mid T0).\ P(T0) + P(R1 \mid T1).\ P(T1)$$
$$= (0.1) \times (0.45) + (0.85) \times (0.55) = 0.5125$$

$$P(T1 \mid R1) = P(R1 \mid T1)\ P(T1) / P(R1) = (0.85) \times (0.55) / 0.5125 = 0.9121$$
$$P(T0 \mid R0) = P(R0 \mid T0)\ P(T0) / P(R0) = (0.9) \times (0.45) / 0.4875 = 0.8307$$
$$P(T1 \mid R0) = 1 - P(T0 \mid R0) = 0.1692$$
$$P(T0 \mid R1) = 1 - P(T1 \mid R1) = 0.0879$$

So we have:

a) Probability that a 0 is received: P(R0) = 0.4875

b) Probability that a 1 was transmitted, given that a 1 was received: P(T1 | R1) = 0.9121

c) Probability that a 1 was transmitted, given that a 0 was received: P(T1 | R0) = 0.1692

d) Probability of an error.

$$P(Error) = P[T1 \cap R0] + P[T0 \cap R1]$$
$$= P(T1 \mid R0)\ .\ P(R0) + P(T0 \mid R1)\ .\ P(R1)$$
$$= P(R1 \mid T0)\ .\ P(T0) + P(R0 \mid T1)\ .\ P(T1)$$
$$= (0.1) \times (0.45) + (0.15) \times (0.55) = 0.1275$$