ECE 313
Homework 9
Due Date: Wednesday, April 17, 2024
Write your name and NetID on top of all the pages. **Show your work to get partial credit.**

Problem 1 – Let N_t be the number of queries arriving in t seconds at a call center. Suppose the inter-arrival times (T) of queries are exponentially distributed with the mean of 0.25 minutes and the query arrivals are independent.

a) Write the pdf of T.

b) Write the pmf of N_t.

c) Find the probability of having more than 4 queries in 10 seconds

d) Find the probability of having fewer than 5 queries in 2 minutes

Problem 2 – Let X be a random variable exponentially distributed with parameter λ. Find the CDF of
   a) $Y = \exp(X)$
   b) $Z = \min(X, 3)$

Problem 3 – Let the joint pdf of X and Y be given by:

$$f(x,y) = e^{-\frac{x}{\alpha}}ye^{-y^2}, for\ x > 0, y > 0$$

where $\alpha \neq 0$. The random variables X and Y are said to have a two-dimensional (or bivariate) normal pdf.
a)   Show that the marginal pdf's of X and Y are:

$$f(x) = \frac{1}{2}e^{-\frac{x}{\alpha}} \qquad and \qquad f(y) = \alpha y e^{-y^2}$$

b)   Find the values of α, for which X and Y are independent.

Problem 4 – Let X_1 and X_2 be two independent random variables exponentially distributed with parameters λ_1 and λ_2.
   a)   Find the pdf of $Z = \min(X_1, X_2)$.
   b)   Find the pdf of $R = \dfrac{X_1}{X_2}$

Problem 5 – Suppose that random variables X and Y are jointly distributed with joint density $f_{X,Y}(x,y) = c(1 + xy)$ for $2 \leq x \leq 3$ and $1 \leq y \leq 2$. $f_{X,Y}(x,y) = 0$ otherwise.
   a)   Find the constant c.
   b)   Find the marginal pdf's of X and Y.