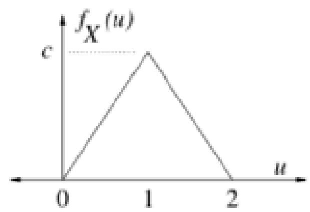
ECE 313
Homework 8
Due Date: April 10, 2024

Write your name and NetID on top of all the pages. **Show your work to get partial credit.**

Problem 1

Suppose X has the pdf shown:



(a) Find the constant c.

(b) Let $\widetilde{X}$ denote the standardized version of X. Thus, $\widetilde{X} = \frac{X-a}{b}$ for some constants a and b so that $\widetilde{X}$ has mean zero and variance one. Carefully **sketch** the pdf of $\widetilde{X}$. Be sure to indicate both the horizontal and vertical scales of your sketch by labeling at least one nonzero point on each of the axes. (Hint: $\text{Var}(X) = \frac{1}{6}$.)

Problem 2

[Covariance for sampling without replacement]
Five balls numbered one through five are in a bag. Two balls are drawn at random, without replacement, with all possible outcomes having equal probability. Let X be the number on the first ball drawn and Y be the number on the second ball drawn.

(a) Find $E[X]$.

(b) Find $\text{Var}(X)$.

(c) Find $E[XY]$.

Problem 3

Suppose $Y = e^X$, where X is an exponentially distributed random variable with parameter λ.

(a) What is $E[Y]$? Give a simple answer depending on λ that is valid for $\lambda > 1$.

(b) What is the support of the pdf $f_Y(v)$ (the set of v for which $f_Y(v) \neq 0$)?

(c) For the set of v that you specified in part (b), find the CDF $F_Y(v)$.

Problem 4

Find a function g so that, if U is uniformly distributed over the interval $[0,1]$, and $X = g(U)$, then X has the pdf:

$$f_X(v) = \begin{cases} 2v & \text{if } 0 \le u \le 1 \\ 0 & \text{else.} \end{cases}$$

(Hint: Begin by finding the cumulative distribution function F_X.)

Problem 5

Prove that the Markov inequality

$$P\{X \ge c\} \le \frac{E[X]}{c}$$

holds with equality if and only if $p_X(0) + p_X(c) = 1$. One way to proceed is to first define the random variable

$$Y_c = X - c\,\mathbb{1}_{\{X \ge c\}}$$

where the *indicator function* $\mathbb{1}_{\{.\}}$ is defined by

$$\mathbb{1}_{\{x \ge c\}} = \begin{cases} 1, & x \ge c \\ 0, & x < c. \end{cases}$$

Then, consider the expected value of Y_c. Note that X is a nonnegative random variable. What does this imply for the random variable Y_c (you will find this useful in your proof)?