ECE 313
Homework 7
Due Date: Wednesday, April, 3, 2024

Write your name and NetID on top of all the pages. **Show your work to get partial credit.**

Problem 1 – Jobs arriving to a compute server have been found to require CPU time T that can be modeled by an exponential distribution with parameter 1/120 ms^{-1}. Assume the number of job requests arriving at a Web server in an interval of t ms, N_t, with parameter $1/120t$. The CPU scheduling discipline is quantum-oriented so that a job not completing within a quantum of 100ms will be routed back to the tail of the queue of waiting jobs.

a) Find the probability that an arriving job is forced to wait until the second quantum using the CDF of T.
b) Of the 800 jobs coming in during a day, how many are expected to finish within the first quantum?
c) For a job to wait until the second quantum, how many jobs should arrive in an interval of 100ms? Solve the question in part b) using the CDF of N_t.

Problem 2 – The lifetime of the memory chips produced by a factory is exponentially distributed with parameter λ = 0.1(years)−1. Suppose John bought a computer with a memory chip produced by this factory and after five years it is still working. What is the conditional probability it will still work for at least three more years?

Problem 3 – The number of errors produced by an operating system in a day is described with the random variable X that has the following pmf:

No. of Failures	0	1	2	3	4	5	6
Probability	0.18	0.28	0.25	0.18	0.06	0.04	0.01

a) Find the expected number of failures in a day.
b) Find the variance of the number of failures in a day.

Problem 4 – Assume the number of students arriving to the class in an interval of t minutes, N_t, is Poisson distributed with mean rate of 2 students per minute. Let T represent the inter-arrival times of students.

a) Write the probability mass function (pmf) of N.
b) What distribution best describes the inter-arrival times of the students? Write the probability density function (pdf) of T.

c) Calculate the expected number of students arriving in a period of 10 minutes.
d) Calculate the expected number of minutes until the arrival of the next student.

Problem 5 – The weekly consumption of gasoline in a factory, in thousands of gallons, is a continuous random variable X having the following probability density function. Find $E[X]$, $E[X^2]$, and $Var[X]$.

$$f(x) = \begin{cases} 0.5(x-1), & 1 < x < 3 \\ 0, & elsewhere \end{cases}$$

Problem 6 – Let the random variable X be uniform on the open unit interval (0, 1). Let $Y = g(X) = -ln(X)$.

a) Find the pdf of Y.
b) Calculate the E[Y] using $f_Y(y)$.
c) Now, calculate the E[Y] using $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$