ECE 313 (Section F)
Homework 6 Solution

Problem 1 –

Let N_t be the number of arriving jobs in interval t, , since it is Poisson distributed with parameter λ=0.3t, its pmf can be written as:

$$P(N_t = k) = e^{-0.2t}\frac{(0.2t)^k}{k!} \quad ,k = 0,1,2,\ldots$$

a) $P(N_{10} = 3) = e^{-0.2x10}\frac{(0.2x10)^3}{3!} = 0.18$

b) $P(N_{20} \leq 10) = \sum_{k=0}^{10} P(\mathrm{N}_{20} = k) = \mathrm{e}^{-4}\sum_{k=0}^{10}\frac{4^i}{i!} \cong 1$

c) $P(2 \leq N_{10} \leq 4) = P(N_{10} = 2) + P(N_{10} = 3) + P(N_{10} = 4) = e^{-2}(\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}) = 0.5413$

d) Let A denote: 3 jobs in the first 10 seconds, B denote: 10 jobs in 20 seconds

P(A|B) = P(B|A)P(A)/P(B)

P(B|A) is the probability of having 10 jobs in 20 seconds, given that 3 jobs arrive in the first 10 seconds. This is equivalent to 7 jobs in the remaining 10 seconds = $P(N_{10} = 7)$ Therefore,

$$P(N_{10} = 3|N_{20} = 10) = \frac{P(N_{10} = 3).(N_{20} = 10)}{P(N_{20} = 10)} = \frac{P(N_{10} = 3).P(N_{10} = 7)}{P(N_{20} = 10)} = 0.003 * 0.18/0.005 = 0.108$$

Problem 2 – $f(x) = \begin{cases} kx^2(1-x^3), & 0 < x < 1 \\ 0, & otherwise \end{cases}$

a) $\int_0^1 f(x)dx = 1 \Rightarrow \int_0^1 kx^2(1-x^3)dx = k\left(\frac{x^3}{3}-\frac{x^6}{6}\right)\Big|_0^1 = k\left(\frac{1}{3}-\frac{1}{6}\right) = \frac{k}{6} = 1 \Rightarrow k = 6$

b) $F(a) = \int_0^a f(x)dx = \int_0^a 6x^2(1-x^3)dx = 6\left(\frac{x^3}{3}-\frac{x^6}{6}\right)\Big|_0^a = (2a^3 - a^6)$

$$F(a) = (2a^3 - a^6),\ \ a \geq 0$$

c) $P\{0.25 < X < 0.5\} = \int_{0.25}^{0.5} f(x)dx = \int_{\frac{1}{4}}^{\frac{1}{2}} 6x^2(1-x^3)dx = 0.203$

d) $P\{0.25 < X < 0.5\} = F(0.5) - F(0.25)$

$$= (2(0.5)^3 - (0.5)^6) - (2(0.25)^3 - (0.25)^6) = 0.203$$

Problem 3 (10 points)–

Let $Y = g(X)$.

$E[g(X)] = \Sigma y p_Y(g(X) = y)$

$E[g(X)] = \Sigma y \Sigma_{i:g(u_i)=y} p_X(u_i)$

$E[g(X)] = \Sigma_i g(u_i) p_X(u_i)$

Problem 4

The expected value of a continuous random variable is calculated by integrating over its PDF

$$E(x) = \int_{-\infty}^{\infty} x * f(x)\, dx$$

Since the PDF is 0 for x<0, we only need to integrate over the non-negative range:

$$E(x) = \int_{0}^{\infty} x * \mu e^{-\mu x}\, dx$$

Therefore, the expected value of the random variable X is:

$$E(x) = [-xe^{-\mu x}]_0^\infty + \int_0^\infty e^{-\mu x}\, dx = \frac{1}{\mu}$$

Problem 5 – To calculate $P(X \geq 4 * 10^6)$ for the normally distributed variable X, we first convert it to a standard normal random variable by subtracting mean (μ) and dividing by standard deviation (σ):

$$\begin{aligned} P(X \geq 4 * 10^6) &= P\left(\frac{X - 5 * 10^6}{5 * 10^5} > \frac{4 * 10^6 - 5 * 10^6}{5 * 10^5}\right) \\ &= P(Z > -2) \\ &= 1 - F_Z(-2) \\ &= F_Z(2) \\ &= 0.9772 > 0.95 \end{aligned}$$

Therefore, the deal will be made.