ECE 313
Homework 6
March 27, 2024

Problem 1 – Assume that the number of jobs arriving to the Blue Waters super-computer in an interval of *t* seconds is Poisson distributed with parameter $\lambda = 0.2t$. Compute the probabilities of the following events:

a) Exactly 3 jobs will arrive during a 10s interval.

b) At most 10 jobs arrive in a period of 20s.

c) The number of job arrivals in an interval of 10s duration is between two and four.

d) Given that 10 jobs arrive in a period of 20s, what is the conditional probability that 3 jobs arrived in the first 10s?

Hint: Use the Bayes theorem to calculate the conditional probability. Note that the probability of 3 jobs arriving in the first 10s, given that 10 jobs arrived in 20s, equals to the probability of 3 jobs arriving in the first 10s and 7 jobs arriving in the second 10s. Also note that the number of arrivals in different time intervals are independent from each other.

Problem 2 – Let *X* be a random variable with probability density function:

$$f(x) = \begin{cases} kx^2(1 - x^3), & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

a) Find the value of the constant *k*.
b) What is the cumulative distribution function of *X*?
c) Find $P\{0.25 < X < 0.5\}$ by using the probability density function.
d) Find the probability in part (c) by using the cumulative distribution function $F_X(x)$.

Problem 3 – Provide a proof for the Law of the Unconscious Statistician (in the discrete case): if X is a discrete-type random variable with probability mass function $p_X(u)$, then, for any real function (a real-valued function of a real variable) g

$$E[g(X)] = \sum_i g(u_i)\, p_X(u_i)$$

Problem 4

Let X be a continuous random variable with the probability density function (PDF):

$$f(x) = \begin{cases} \mu e^{-\mu x}, & 0 \leq x \\ 0, & otherwise \end{cases}$$

where $\mu > 0$ is a constant. Find the expected value of the random variable X.

Problem 5 – Lifetimes of VLSI chips manufactured by a semiconductor manufacturer are approximately normally distributed with $\mu = 5x10^6$ h and $\sigma = 5x10^5$ h. A computer manufacturer requires that at least 95% of a batch should have a lifetime greater than $4x10^6$h. Will the deal be made?