ECE 313
Homework 5
Due Date: Wednesday, March 20, 2024

Show your work to get partial credit.

Problem 1 – A company manufactures CD-ROMs with a mean diameter of 15cm and a standard deviation of 5mm. We can approximate the distribution of the diameters by a normal distribution. After manufacturing, the CDs which have a diameter between 14.2cm and 15.8cm are packed for sales; the rest are discarded. What percent of the CDs

a) will have a diameter greater than 15.3cm?
b) will be discarded?

Problem 2 – Suppose two fair dice are rolled. Let X be the minimum of the two numbers showing. For example, if a 2 shows on the first die and a 5 shows on the second, then X = 2. That is, X((2, 5)) = 2. In general, X((i, j)) = min{i, j} for (i, j) $\in \Omega$. Find the numerical values of the following quantities (show your work to get credit):

a) Calculate and draw the pmf of X.
b) Draw the cumulative distribution function (CDF) of X.
c) $P\{X \leq 1\}$
d) $P\{X < 1\}$
e) $P\{X \geq 6\}$
f) $P\{X = 6\}$
g) $P\{X > 6\}$
h) $P\{3 \leq X \leq 6\}$
i) $P\{|X - 3| \leq 0.1\}$.

Problem 3 – From previous inspections, in every 100 memory chips produced by a plant 6 are known to be defective. A sample of 10 memory chips is drawn randomly from each week's production. If X is the number of defective chips found in the weekly samples, find:

a) The pmf of X. What is the distribution of X?
b) The probability that none of the chips is defective.
c) The probability that exactly one chip is defective.
d) The probability that more than 2 chips are defective.

Problem 4 – The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$$

a) Find P{X > 20}.

b) What is the cumulative distribution function of X?

c) What is the probability that, of 5 such types of devices, at least 2 will function for at least 15 hours? What assumptions are you making?

Problem 5 – A datacenter has *n* disk drives. A software engineer has written a program to inspect the disks at the end of each day. Each test takes about 1 minute to run. Suppose that the disk drives are equally likely to be defective and only one of them is defective in any given day. In order to optimize the time, the program chooses the disk drives independently and at random order and tests them until finds the defective one. Let *N* be the number of minutes required to find the defective disk drive. Determine the probability mass function (pmf) of *N*:

a) if the program doesn't save the results of previous tests (The program might test a previously tested disk drive again)
b) if the previously tested disk drives are marked by the program and are not tested again.

Problem 6– Consider the following program segments consisting of a *while* loop:

```
int  B = randInt();
//   randInt() is a random integer generator that returns an integer between 1 and 10.
//   All integers are equally likely to be generated.
```

i.
```
while (B ≥ 7){
     execute S;
     B = randInt();
}
```
ii.
```
do{
     execute S;
     B = randInt();
}while (B ≥ 7);
```

In the first program segment, the loop is taken while the Boolean expression ($B \geq 7$) is TRUE. Variable B gets a new random value after each execution of S. In the second program segment, the condition is tested after the execution of the loop.
If the successive executions of the loop are independent from each other, then let *X* be the number of times the body (or the statement-group S) of the loop is executed.

a) Let p be the probability of the loop condition ($B \geq 7$) being **TRUE**. Find the value of p.

For each of the program segments **i** and **ii**,

b) Determine the set of values that the random variable X might take.
c) Derive the probability mass function (pmf) of X and determine the type of distribution, **in terms of p**.

Hint: In one of the above program segments, X has a geometric distribution and in the other case it has a modified geometric distribution.