ECE 313
Homework 4
Due Date: Wednesday, March 13th

Write your name and NetID on top of all the pages. Show your work to get partial credit.

Problem 1 – For each of the following cases, determine whether the random variable is discrete or continuous. For the discrete random variables: (i) Describe the probability mass function (pmf) by finding the set of values that the random variable might take and their respective probabilities (The solution to Parts (a) and (b) are given as examples for you).

a) T is the lifetime of a light bulb.
Solution: T is a continuous random variable.

b) A certain manufacturing firm produces chipsets that are non-defective with the probability of p = 0.9. A quality control crew randomly picks chipsets and tests them for defects. He is asked to repeat this process until the first defective product is found.

X is the number of defective chipsets in the sample of 10 randomly chosen chips

N is the number of trials until the first defective chipset is found

Solution: X is a discrete random variable.
It takes values of {0, 1, 2, 3, ..., 10} with the following probabilities:
$P\{X=0\} = p(0) = \binom{10}{0}(0.9)^{10}(0.1)^{0} \approx 0.3487$
$P\{X=1\} = p(1) = \binom{10}{1}(0.9)^{9}(0.1)^{1} \approx 0.3874$
$\vdots$
$P\{X=10\} = p(10) = \binom{10}{10}(0.9)^{0}(0.1)^{10} = (0.1)^{10}$
In general $P\{X=k\} = p(k) = \binom{10}{k}(0.9)^{10-k}(0.1)^{k}$ k = 0,1,...,10

N is also a discrete random variable.
It takes values of {1, 2, 3, ...} with the following probabilities:
$P\{N=1\} = p(1) = p = 0.1$
$P\{N=2\} = p(2) = (1-p)p = (1-0.1)(0.1) = 0.09$
$P\{N=3\} = p(3) = (1-p)^2 p = (1-0.1)^2(0.1) = 0.081$
$\vdots$
In general $P\{N=n\} = p(n) = (1-p)^{n-1}p$ n = 1, 2, ...

c) Let Y represent the difference between the number of heads and the number of tails obtained when a fair coin is tossed 3 times.

d) L is the lifetime of a valve in a randomly selected cooling cabinet.

e) An amateur mind game player has accepted a challenge from a computer program. The challenge consists of three rounds (with different games for each round) and the player is awarded \$1,000 for each round that the player has won. In addition, the final winner (who wins more than two out of the three rounds) is awarded additional \$3,000. A preliminary analysis has shown that the possibility of the player winning the computer program is 0.55 for the first round, 0.7 for the second round and 0.4 for the third round. Let X be a random variable on the total award that the player can win. Assume that all three rounds are played regardless of the results.

f) Assume we have weather forecasting application with a prediction accuracy of 85%. Let W be a random variable representing the number of days in a week that the application successfully predicts the weather.

g) Let A represent an analog signal received by an analog to digital converter.

h) A multiple-choice exam consists of 5 questions, each question containing 3 possible answers. Let X be a discrete random variable representing the number of correct answers that a student would get just by guessing.

Problem 2 – Consider the random variable X with pmf $P(X = i) = 2^{-i}$ for $i \geq 1$.

a) Sketch the pmf (given above) for X.

b) Show that the given pmf is a valid pmf. Hint: what condition needs to be satisfied for a function to be a valid pmf?

c) Calculate $P(X \leq 4)$.

d) Calculate $P(X > 4)$.

e) Calculate $P(X < 1)$.

f) Calculate $P(|X-5| \leq 0.1)$.

g) Evaluate the following expression:

$$\sum_{k=6}^{\infty} P(X = k)$$

Problem 3 – For each of the following, please describe whether it is a valid CDF or not. Justify your answers.

a $F(x) = \begin{cases} 0, & if\ x < 0 \\ 4x^4 - 3x^2 & if\ 0 \le x < 1 \\ 1 & if\ x \ge 1 \end{cases}$

b $F(x) = \begin{cases} 0, & if\ x < 0 \\ 0.5 + e^{-x} & if\ 0 \le x < 3 \\ 1 & if\ x \ge 3 \end{cases}$

c $F(x) = \begin{cases} 0, & if\ x < -1 \\ x^2 & if -1 \le x < 1 \\ 1 & if\ x \ge 1 \end{cases}$

Problem 4 –

1) The length of time a teacher can write on the blackboard without breaking the chalk is the random variable T (measured in minutes) with the following pdf:

$$f_T(t) = \begin{cases} (0.05)e^{-(0.05)t}, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

a) Determine the CDF F_T for T.

b) If the teacher has not broken the chalk after 10 minutes, what is the probability that she will break the chalk in the next 20 minutes?

Problem 5 –Five distinct numbers are randomly distributed to players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let X denote the number of times player 1 is a winner. Find P{X = i}, i = 0, 1, 2, 3, 4.