ECE 313
Homework 3 Solution

Problem 1 (10 points)–

Given that occurrence of different kinds of defects are independent from each other, we have:

$$P(D)$$
$$= P(A) + P(B) + P(C) - P(A)P(B) - P(B)P(C) - P(A)P(C) + P(A)P(B)P(C)$$

= 0.04 + 0.08 + 0.1 – 0.04*0.08 – 0.08*0.1 – 0.04*0.1 + 0.04*0.08*0.1

= 0.22 – 0.0032 – 0.008 – 0.004 + 0.00032 = 0.20512

Thus:

a) $P(not\ defective) = P(\overline{D}) = 1 - P(D) = 1 - 0.20512 = 0.79488$ (3 points)

b) $P(defective) = P(D) = 0.20512$ (3 points)

$$P(one\ kind\ of\ defect) = P(B)P(C) - P(A)P(B)P(C)$$
$$= 0.08 \times 0.1 - 0.00032 = 0.00768$$

c)

$$P(A, B, C \mid defective) = \frac{P(A, B, C \ \cap \ defective)}{P(defective)}$$
$$= \frac{0.00032}{0.20512} = 0.00156$$

(4 points)

Problem 2 (12 points) –

$$P(System\ working) = P(System\ working \mid C3\ working).P(C3\ working) + P(System\ working \mid C3\ not\ working).P(C3\ not\ working)$$

(6 points)

$P(System\ working \mid C3\ working) = 1 - (1 - R_4)(1 - R_5R_6)$ **(2 points)**

$P(System\ working \mid C3\ not\ working) = (1 - (1 - R_1)(1 - R_2))R_4$ **(2 points)**

$$P(System\ working) = \big(1 - (1 - R_4)(1 - R_5R_6)\big)R_3 + (1 - (1 - R_1)(1 - R_2))R_4(1 - R_3)$$

(2 points)

(They can also use ‘C’ instead of ‘R’ in the answers)

Problem 3 – (8 POINTS)

Each question worth 2 points. As long as Students write procedure, they gain half credits. If the answer is correct, they gain full credits.

(a) $\binom{12}{0}\ (0.999)^{12} \ = \ (0.999)^{12} \ = \ 0.988.$

(b) $\binom{12}{1}\ (0.999)^{11}\ (0.001) \ = \ 0.011869.$

(c) $\sum_{i=1}^{12} \ \binom{12}{i}(0.999)^{12-i}(.001)^i = 1 - \binom{12}{0}\ (0.999)^{12} = 0.012.$

(d)

$$\begin{aligned}\binom{12}{2}\ (0.999)^{10}\ (0.001)^2 &= \frac{12!}{10!\ 2!}\ (0.999)^{10}\ (0.001)^2 \\ &= 66\ (0.999)^{10}\ (0.001)^2 \\ &= 0.000065.\end{aligned}$$

Problem 4 (10 points) –

a) **(6 points)** Let C_1 and C_2 denote the states of the two clocks. Let each clock be in state G (good) or $\bar{G}$ (bad). We are asked to compute: $P(C_2 = G \mid C_1 = G)$.

We assume that the container is equally likely to be from either of the factories, i.e. $P(A) = P(B) = 0.5$. From the problem statement, it is reasonable to assume that each clock made in factory A may turn faulty with probability 0.01, independent of the other clocks made in A. Same can be said about B, i.e.

$P(C_2 = \bar{G} \mid C_1 = G, A) \ = \ P(C_2 = \bar{G} \mid A) = 0.02$ **(1 point)**
$P(C_2 = \bar{G} \mid C_1 = G, B) \ = \ P(C_2 = \bar{G} \mid B) = 0.01.$ **(1 point)**

Notice that,
$P(C_1 = G) = P(A)\ P(C_1 = G \mid A) + P(B)\ P(C_1 = G \mid B) = 0.5\ (0.98) + 0.5\ (0.99).$
(1 points)

Now $P(C_2 = G \mid C_1 = G) \ = \dfrac{P(C1 = G, C2 = G)}{P(C1 = G)}$ (1 points)

$= \dfrac{P(A)\ P(C2 = G, C1 = G \mid A) + P(B)\ P(C2 = G, C1 = G \mid B)}{P(C1 = G)}$ (1 pts)

$= \dfrac{P(A)P(C1 = G|A)P(C2 = G|A) + P(B)P(C1 = G|B)P(C2 = G|B)}{P(C1 = G)}$

$= (0.98^2 + 0.99^2) / (0.98 + 0.99)$ **(1 pts)**

b) **(4 points)** $P(A \mid C1 = G, C2 = G) = \frac{P(A,\ C1 = G,\ C2 = G)}{P(C1 = G,\ C2=G)}$

$$= \frac{P(\ C1 = G,\ \ C2 = G|A)\ P(A)}{P(C1 = G,\ \ C2=G)}$$

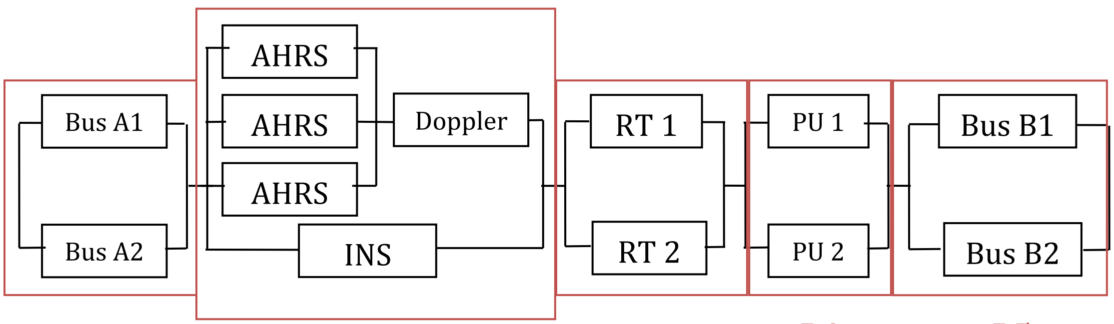
$$= \frac{P(C1 = G|A)\ \ P(C2 = G|A)\ \ P(A)}{P(C2=G\ |C1 = G)\ P(C1 = G)}$$

$$= \frac{0.98 * 0.98 * 0.5}{(0.98*0.98+0.99*0.99)*0.5} = 0.4949$$

Problem 5(10 points) –

1) Based on the problem description, we know that for the system to work, we need to have both the Buses A and B, the remote terminal (RT), the processing unit (PU), as well as the navigation system. The navigation system can work by either INS working or one AHRS and Doppler working.

2) The reliability block diagram of the system can be drawn as follows:



3) At the high-level we can consider the system as the series of 5 main components shown above: Bus A, INS/AHR, RT, PU, and Bus B. So we can write the high-level reliability formula as:

$$R_{System} = R1.R2.R3.R4.R5$$

Then we can write the reliability formula for each of R1, R2, R3, R4, and R5 blocks to get the reliability of system based on the reliability of the components. R1, R3, R4, and R5 blocks are each a parallel system, while R2 is a series-parallel system itself:

$$\begin{aligned} R_{System} = &[1-(1-R_{BusA})^2]. \\ &\{1-[(1-R_{INS}).(1-R_{DOP}.(1-(1-R_{AHRS})^3))]\}. \\ &[1-(1-R_{RT})^2]. \\ &[1-(1-R_{PU})^2]. \end{aligned}$$

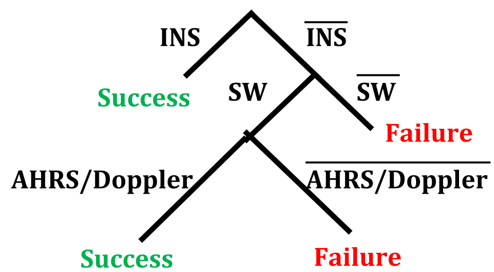
$$[1-(1-R_{BusB})^2]$$

Finally, we just plug-in the reliability numbers given for each component from the table:

$$\begin{aligned} R_{System} &= [1-(1-0.88)^2]. \\ &\{1-[(1-0.85).(1-0.9(1-(1-0.88)^3))]\}. \\ &[1-(1-0.95)^2]. \\ &[1-(1-0.92)^2]. \\ &[1-(1-0.88)^2] \\ &= 0.94811 \end{aligned}$$

Hint: You can check your answer by typing the formula in http://www.wolframalpha.com.

4) The tree diagram for block R2 (INS/AHRS/Doppler) when the switch (SW) is not perfect is as follows:



So the failure for block R2 can be written as:

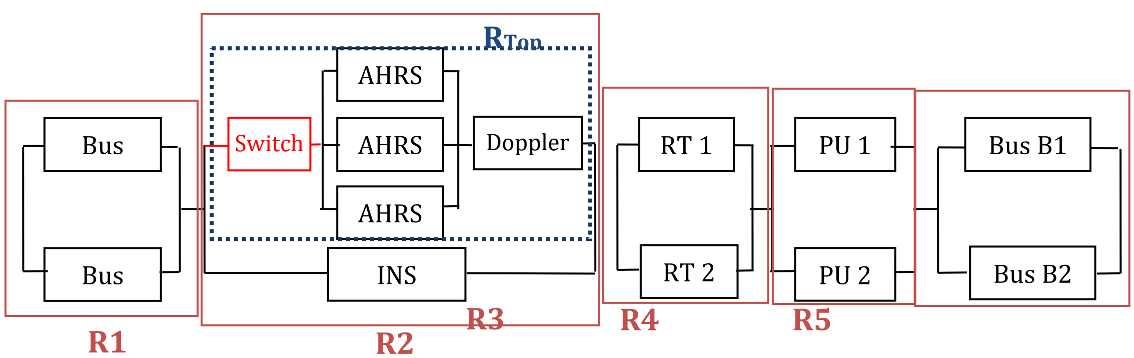
$$1-R2 = 1-[R_{INS}+(1-R_{INS}).R_{SW}.(1-R_{DOP}.(1-(1-R_{AHRS})^3))]$$

If we substitute $(1-R_{DOP}.(1-(1-R_{AHRS})^3))$ with R_{DOP_AHRS}:

$$\begin{aligned} 1-R2 = \; & 1-[R_{INS}+(1-R_{INS}).R_{SW}.R_{DOP_{AHRS}}] \\ &= 1-R_{INS}+(R_{INS}.R_{SW}.R_{DOP_{AHRS}})-R_{SW}.R_{DOP_{AHRS}} \\ &= (1-R_{INS})(1-R_{SW}.R_{DOP_{AHRS}}) \end{aligned}$$

$$=> \qquad R2 = 1-(1-R_{INS})(1-R_{SW}.R_{DOP_{AHRS}})$$

From the formula, we can see that the Switch will be in series with AHRS and Doppler, and in parallel with INS. So the reliability block diagram for the whole system will be changed as follows:



And the formula for the reliability of system will be changed to:

$$R_{System} = R1.R2.R3.R4.R5$$
$$R2 = 1 - (1 - R_{INS})(1 - R_{Top})$$
$$R_{Top} = R_{SW}.R_{DOP}.(1 - (1 - R_{AHRS})^3)$$
$$\boldsymbol{R_{System}} = [1 - (1 - R_{BusA})^2]. \left\{1 - \left[(1 - R_{INS}).\left(1 - R_{SW}.R_{DOP}.(1 - (1 - R_{AHRS})^3)\right)\right]\right\}. [1 - (1 - R_{RT})^2].[1 - (1 - R_{PU})^2].[1 - (1 - R_{BusB})^2]$$
$$= [1 - (1 - 0.88)^2]. \left\{1 - \left[(1 - 0.85).\left(1 - 0.9.0.9(1 - (1 - 0.88)^3)\right)\right]\right\}. [1 - (1 - 0.95)^2].[1 - (1 - 0.92)^2].[1 - (1 - 0.88)^2] = 0.935$$