ECE 313
Homework 2

Due: Wednesday, January 31

Problem 1

A Petascale Computing Facility, which houses a supercomputer, needs chilled water cooling to keep the system operating within an acceptable temperature range. Figure 1, is an overview of the Liebert XDP unit which cools the compute cabinets of the supercomputer. As shown in the figure below, primary and backup pumps are used to maintain the flow of chilled water. An iCOM controller/switch monitors the status of the pumps and switches from primary to backup upon detecting a pump failure.

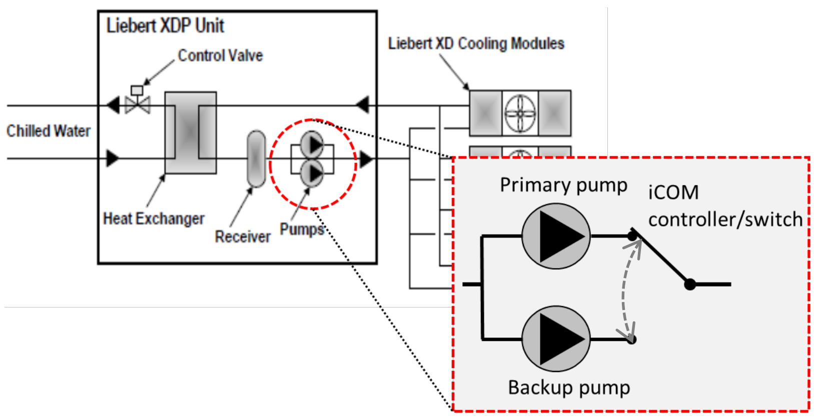


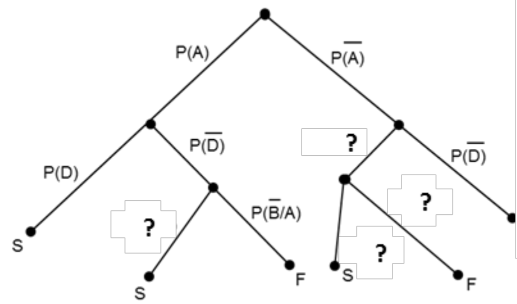
Figure 1. Overview of a Liebert XDP Unit

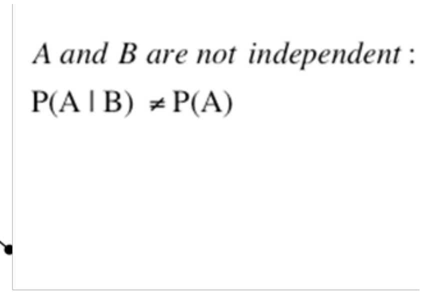
We define the following events:

A = "Primary pump functions correctly."
$\bar{A}$ = "Primary pump fails to function correctly."
B = "Backup pump functions correctly."
$\bar{B}$ = "Backup pump fails to function correctly."
D = "iCOM detects pump failure correctly."
$\bar{D}$ = "iCOM fails to detect pump failure
or raises a false alarm while the primary pump is operational."

Assume that event pairs A and D as well as B and D are independent, but events A and B are dependent.

a) Complete the following tree by replacing the question marks with probability expressions. S stands for success and F represents a failure. Each path from the root to a leaf in the tree represents one of the ways that system would fail or succeed.





b) Derive an expression for the failure probability of the pump system highlighted in Figure 1. Make sure to simplify the expression.

Problem 2 – Prove or give counterexamples to the following statements:

a) If E is independent of F, and E is independent of G, then E is independent of $F \cup G$.

b) If E is independent of F, and E is independent of G, and $F \cap G = \emptyset$, then E is independent of $F \cup G$.

Problem 3 – A total of 44 percent of the voters in a certain city classify themselves as independents, whereas 26 percent classify themselves as liberals, and 30 percent classify themselves as conservatives. In a recent local election, 55 percent of the independents, 38 percent of the liberals, and 32 percent of the conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that he or she is:

a) An independent

b) A liberal

c) A conservative

d) What fraction of voters participated in the local election?

Problem 4 – An AND gate takes two inputs A and B that are independent. P (A = 0) = P (B = 0) =0.6, P (A = 1) = P (B = 1) = 0.4. The inputs are gated to a clock so that they only change on the falling edge of the clock signal. On a given falling edge of the clock, what is the probability that the output C of the AND gate switches (i.e., the output was previously a 0 and transitioned to a 1 when the new inputs A and B are applied, and vice versa)? Note that both inputs A and B can change on the falling clock edge.

Problem 5 – Box A has 4 white balls and 5 black balls. Box B has 3 white balls and 4 black balls. A ball is drawn at random from Box A and transferred to Box B without looking at the ball's color. Then a ball is drawn at random from Box B. What is the probability that the ball drawn from Box B is black?

Problem 6 – How can 20 balls, 10 white and 10 black, be put into two urns so as to maximize the probability of drawing a white ball if an urn is selected at random and the ball is drawn at random from it?