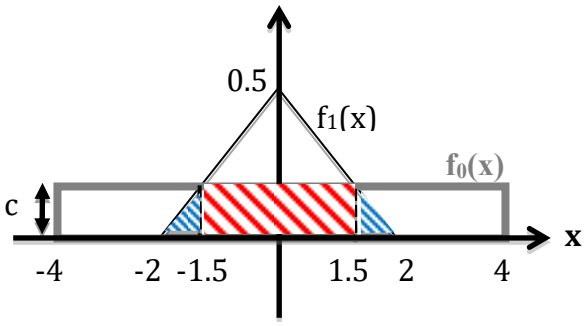
ECE 313
Homework 12

Due Date: Wednesday, May 22, 2024

Write your name and NetID on top of all the pages. **Show your work to get partial credit.**

Problem 1 – As shown in the following figure, an observation X is drawn from a triangular distribution $f_1(x)$ with range [-2, 2] if hypothesis H_1 is true, and is drawn from a uniform distribution $f_0(x)$ with range [-4, 4] if hypothesis H_0 is true. Assume that the prior probability $P(H_0)$ is 0.8.



a) Find the value of c.
b) Determine the ML decision rule.
c) Determine the false alarm and miss detection probabilities for the ML decision rule.
d) Determine the MAP decision rule.
e) Determine the false alarm and miss detection probabilities for the MAP decision rule.
f) Determine the error probability for both the ML and MAP decision rules.

Problem 2 – Suppose that a store has two types of oranges (X) – clementines ($X = 0$) and mandarins ($X = 1$). Suppose $X \sim \text{Bernoulli}\left(\frac{1}{4}\right)$. The weight of an orange denoted by Y is a continuous and observable random variable. The conditional probability density of Y is given by the Laplace distribution:

$$f_{Y|X}(y \mid X = k) = \begin{cases} \frac{1}{2}\exp(-|y|), & k = 0 \\ \frac{1}{6}\exp\left(-\frac{|y-3|}{3}\right), & k = 1 \end{cases}$$

a) Compute the likelihood ratio $\Lambda(Y)$.
b) What are the values of π_0 and π_1 for this problem?
c) Determine the MAP decision rule to identify the type of an orange from its weight. Express the decision rule in a simplified form.
d) Under what circumstances would the MAP decision rule the ML decision rule be the same for this problem?

Problem 3 – Given the following likelihood matrix, specify the ML and MAP decision rules and find $p_{false-alarm}$, p_{miss}, and p_e for both of them. Assume that: $\pi_0 = 0.3$.

	X = 0	X = 1	X = 2	X = 3	X = 4
H_1	0.00	0.10	0.24	0.34	0.32
H_0	0.08	0.15	0.31	0.31	0.15

Problem 4 – We are interested in detecting Denial of Service (DOS) attacks to a Web server based on the incoming traffic that is observed at a given second.

- Suppose that usually the number of requests, *X*, arriving at the Web Server from one client in an interval of 1 second is Poisson distributed with parameter $\lambda = 3$.
- In case of a DOS attack, an attacker sends a total of 100 malicious requests per second, from which *X* would successfully pass the firewall and arrive to the Web Server. Suppose the probability that a malicious request passes through the firewall and arrives to the server is $p = 0.04$.

A decision rule is needed to decide, based on observation of *X,* whether an attack occurred (hypothesis H_1) or not (hypothesis H_0).

a) Write the conditional pmf of observation *X* given hypotheses H_0.

b) Write the conditional pmf of observation *X* given hypotheses H_1. (**Hint:** What distribution best describes the number of malicious requests that successfully arrive to the Web Server?)

c) Describe the ML decision rule in the form of likelihood ratio tests (LRT).

(**Hint:** For which values of *X* an attack (H_1) is declared by the ML decision rule?)

d) Describe the MAP decision rule in the form of likelihood ratio tests (LRT), under the assumption that receiving normal requests is a priori three times more likely than receiving attacks (i.e. $\frac{\pi_0}{\pi_1} = 2$).

e) Find $p_{false-alarm}$, p_{miss}, and p_e for each of the ML and MAP decision rules.

f) Compare the performance of the ML or MAP decision rules.