ECE 313
Homework 11
Due Date: Wednesday, May 15, 2024

Write your name and NetID on top of all the pages. **Show your work to get partial credit.**

Markov Inequality

Problem 1 – Suppose that the average execution time of a job submitted to a computing node at the Blue Waters computing center is known to be 4.39 seconds. We classify a job as **trivial** if it takes less than 1 second to finish, **moderate** if it takes between 1 and 5 seconds to be done, and **complex** otherwise.
Let X be the random variable representing the execution time of a job submitted to the node.

a) Obtain a bound on the probability that a submitted job request is a complex job.
b) Obtain a bound on the probability that a submitted request is not a trivial job.
c) Now, assume that we know X is exponentially distributed with mean 4.39 seconds. Use the information about distribution of X to recalculate the two bounds in parts a and b.

Law of Large Numbers

Problem 2 – Suppose three fair coins are tossed 1000 times. Let X be number of heads showing on each trial.

a) Find the distribution, mean, and variance of X.
b) What is a rough approximation to the total number of heads showing in 1000 trials (coin tosses)? Justify your answer using the law of large numbers.

Central Limit Theorem

Problem 3 – Let X be a random variable representing the lifetime of a specific type of batteries used in an electronic device, with mean 40 hours and standard deviation 20 hours. Assume that you have a stockpile of 25 batteries and after each battery failure you replace it with a new one. If the lifetimes of batteries are independent, approximate the probability that the total usage time of the electronic device exceeds1100 hours.

(**Hint:** The central limit theorem applies to both the sum and average of a sequence of independent and identically distributed random variables).

Achieving potential in a class

Problem 4 - (The following is roughly based on the ECE 313 grading scheme, ignoring homework scores and the effect of partial credit.) Consider a class in which grades are based entirely on midterm and final exams. In all, the exams have 100 separate parts, each worth 5 points. A student scoring at least 85%, or 425 points in total, is guaranteed an A score. Throughout this problem, consider a particular student, who, based on performance in other courses and amount of effort put into the class, is estimated to have a 90% chance to complete any particular part correctly. Problem parts not completed correctly receive zero credit.

(a) Assume that the scores on different parts are independent. Based on the LLN, about what total score for the semester are we likely to see?
(b) Under the same assumptions in part (a), using the CLT (without the continuity correction, to be definite) calculate the approximate probability the student scores enough points for a guaranteed A score.
(c) Consider the following variation of the assumptions in part (a). The problem parts for exams during the semester are grouped into problems of four parts each, and if X_i and X_j are the scores received on two different parts of the same problem, then the correlation coefficient is $\rho_{X_i,X_j} = 0.8$. The scores for different problems are independent. The total score for problem one, for example, could be written as $Y_1 = X_1 + X_2 + X_3 + X_4$, where Xi is the score for the i th part of the problem. Find the mean and variance of Y_1.
(d) Continuing with the assumptions of part (c), the student takes a total of 25 problems in the exams during the semester (with four parts per exam). Using the CLT (without the continuity correction, to be definite) calculate the approximate probability the student scores enough points for a guaranteed A score.