University of Illinois at Urbana-Champaign
Dept. of Electrical and Computer Engineering

ECE 120: Introduction to Computing

Sparse Representations, Bit Errors, and Error Detection

Representations Must be Unambiguous

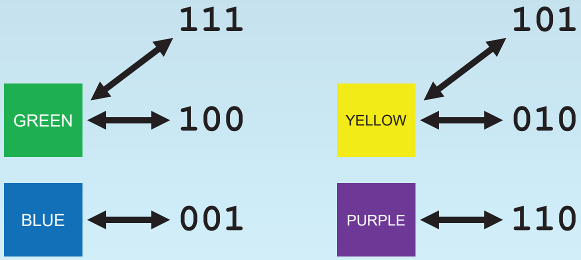
Recall requirements for a representation…

A bit pattern represents **at most one thing**.


GREEN ⟷ 100
BLUE ⟷ 001
GOOD
GREEN ⟷ 100
BLUE ⟷ 100
BAD

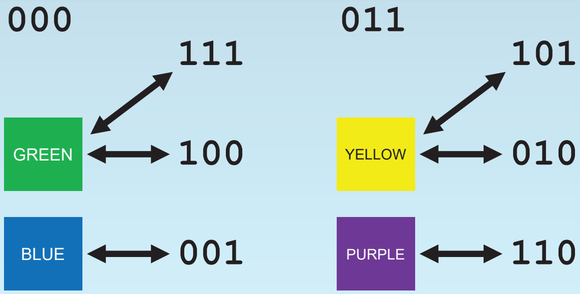
But One Thing Can Be Represented by Several Patterns

For example, **100** and **111**
can both represent **GREEN**.


GREEN ⟷ 100
GREEN ⟷ 111
BLUE ⟷ 001
YELLOW ⟷ 010
YELLOW ⟷ 101
PURPLE ⟷ 110

And Some Patterns May Represent Nothing

Bit patterns **000** and **011** represent no color.


000
011
GREEN ⟷ 100
GREEN ⟷ 111
BLUE ⟷ 001
YELLOW ⟷ 010
YELLOW ⟷ 101
PURPLE ⟷ 110

Example: Binary-Coded Decimal (BCD)

BCD maps **decimal digits** into 4-bit patterns.

Each pattern is the binary value of the digit.

Six patterns are unused ($2^4 - 10 = 6$).

0 ⟷ 0000
1 ⟷ 0001
2 ⟷ 0010
3 ⟷ 0011
4 ⟷ 0100

5 ⟷ 0101
6 ⟷ 0110
7 ⟷ 0111
8 ⟷ 1000
9 ⟷ 1001

1010
1011
1100
1101
1110
1111

Unused Bit Patterns Can Be Used to Detect Errors

Bit patterns on the right have no meaning.

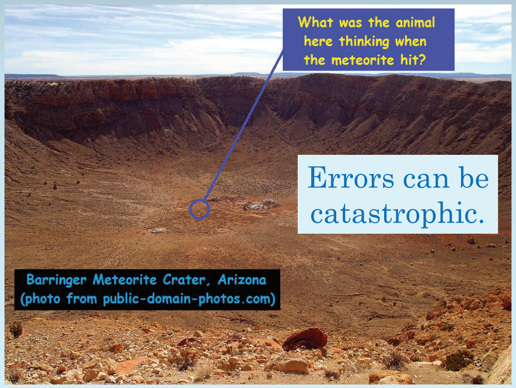
Seeing one of these patterns means that **an error has occurred**.

0 ⟷ 0000
1 ⟷ 0001
2 ⟷ 0010
3 ⟷ 0011
4 ⟷ 0100

5 ⟷ 0101
6 ⟷ 0110
7 ⟷ 0111
8 ⟷ 1000
9 ⟷ 1001

1010
1011
1100
1101
1110
1111

But What Exactly is an "Error?"



What was the animal here thinking when the meteorite hit?

Errors can be catastrophic.

Barringer Meteorite Crater, Arizona
(photo from public-domain-photos.com)

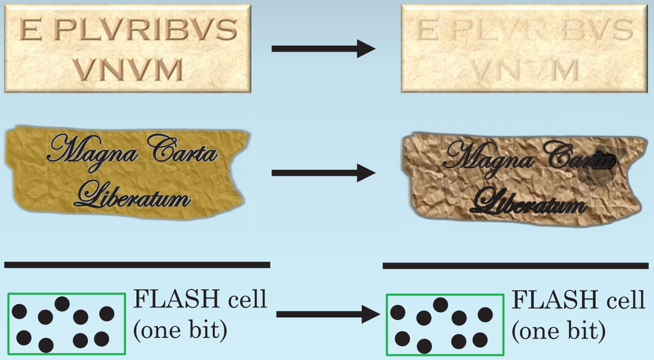
We Need to Choose a Model for Errors

Cannot handle "all" errors.

- Catastrophes are always possible.
- But catastrophes are (hopefully) uncommon!

Let's focus on frequent types of errors.

Errors are Not a New Phenomenon



Our Class Uses Bit Flips as an Error Model

Previous slide showed **erasures**,
in which a character is missing.

our class assumes
- a more difficult type of error
- one character is replaced by another

with binary representations
- these are **bit flips**
- **0 becomes 1, or 1 becomes 0**

Examples from English: Erasures vs. Errors

·hat ·oes t·is ·ay·

What does this say?

Than foen thit xay.

That goes this way.

Assume Small Numbers of Independent Bit Flips

Bit flip error model: 0 becomes 1, 1 becomes 0

Each bit flips
- independently of all others
- with some low probability (call it **p**)

For **N** bits
- chance of one error is $\mathbf{Np}(1 - \mathbf{p})^{N-1}$
- chance of two errors is $\frac{1}{2}\,\mathbf{N(N-1)p}^2(1 - \mathbf{p})^{N-2}$

In practice, **Np** << 1 (**Np** is much less than 1) so
chance of two errors << chance of one error

Example: 2-out-of-5 Code

A 2-out-of-5 code maps
decimal digits into 5-bit patterns.

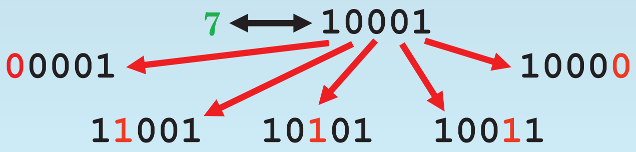
Each pattern has **exactly two 1 bits**.

1	⟷	00011	**6**	⟷	01100
2	⟷	00101	**7**	⟷	10001
3	⟷	00110	**8**	⟷	10010
4	⟷	01001	**9**	⟷	10100
5	⟷	01010	**0**	⟷	11000

Claim: Can Always Detect a Single Bit Error

Thought experiment:

What happens if a single bit flips in a digit coded using a 2-out-of-5 code?



None of these patterns means anything, so
we can always detect the error!