

ECE 120: Introduction to Computing

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Sum-of-Products (SOP) Form is Quite Comm	ion	Product-of-Sum	us (POS) Form is Also Con	nmon
sum-of-products (SOP) a sum (OR) of products (AND) of literals		<b>product-of-sums</b> a production of sums of literal	t (AND) (OR)	
examples: <b>AB + BC</b> , <b>AB' + C + A'C'D'</b> , but NOT <b>A(B + C) + D</b>		`	+ B)(B + C), + B')C(A' + C' + D'), ; NOT (A + BC)D	
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Canonical Forms Allow Easy Comparison, But Are Too Big

#### canonical SOP

a sum of minterms; the expression produced by the logical completeness construction

canonical POS

a sum of maxterms

## What does canonical mean?

**Unique** (if we assume an ordering on variables).

Too many terms to be of practical value.

# Do You Know Mathematical Implication?

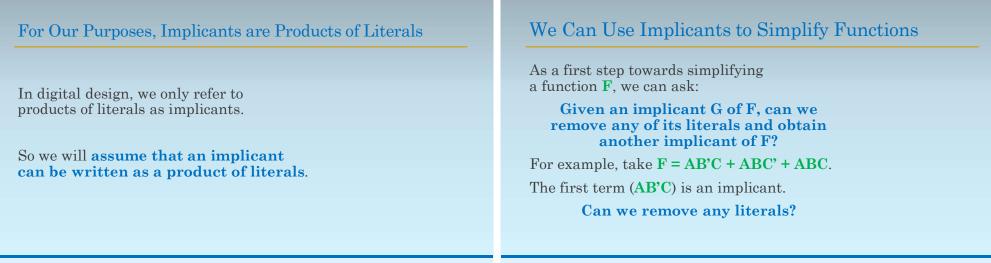
What does A→B mean? A implies B. In other words: if A is true, B is also true.

What if A is false? In that case, is A→B true or false? If A is false, A→B is true.

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So the Following Odd Statements are True All <b>purple elephants</b> can fly. (X is a <b>purple elephant</b> → X can fly.)	One Function Can Imply Another A function <b>G</b> is an implicant of a second function <b>F</b> iff <b>G</b> operates on the same variables as <b>F</b> and $\mathbf{G} \rightarrow \mathbf{F}$ .
<pre>Students who score above 125% in ECE120 fail the class. (X scored above 125% → X fails.)</pre>	In other words, every row • with an output of 1 in <b>G</b> 's truth table • also has an output of 1 in <b>F</b> 's truth table.
In both, <b>the premise is false for any X</b> , so the <b>implications are true</b> .	0 rows in <b>G</b> 's truth table do not matter.
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Try to Remove Each Literal to Find Only AC Implies F	We Remove as Many Literals as We Can
Start from AB'C and try to remove each literal.ABCFB'CACAB'00000000B'C is not an implicant.010000O1100000AC is an implicant.100000AC is an implicant.101111AB' is not an implicant.111111111010011111111	So we can simplify <b>F</b> by replacing <b>AB'C</b> with <b>AC</b> : $\mathbf{F} = \mathbf{AC} + \mathbf{ABC'} + \mathbf{ABC}$ Checking the second term ( <b>ABC'</b> ), we find that we can eliminate <b>C'</b> to obtain: $\mathbf{F} = \mathbf{AC} + \mathbf{AB} + \mathbf{ABC}$ In the third term ( <b>ABC</b> ), we can eliminate <b>B</b> or <b>C</b> , but not both. Let's pick <b>B</b> . $\mathbf{F} = \mathbf{AC} + \mathbf{AB} + \mathbf{AC}$
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Prime Implicants Have a Minimal Number of Literals

### $\mathbf{F} = \mathbf{A}\mathbf{C} + \mathbf{A}\mathbf{B} + \mathbf{A}\mathbf{C}$

But now we have a duplicate term, which we can eliminate to arrive at a simple form for  ${\bf F}:$ 

### $\mathbf{F} = \mathbf{AC} + \mathbf{AB}$

We can remove no more literals.

One more definition: An implicant **G** of **F** is a **prime implicant of F** iff **none of the literals in G can be removed** to produce other implicants of **F**.

AB and AC are prime implicants of F.

# To Simplify, Write Function as a Sum of Prime Implicants

The conclusion is obvious:

To simplify a function F, write it as a sum of prime implicants.

Enjoy the algebra.

Good luck!

(Next time, we'll develop a graphical tool that lets us skip the algebra.)

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