ECE 120 Second Midterm Exam
Spring 2016

Tuesday, March 15, 2016

Name: SOLUTIONS

NetID: ____________

Discussion Section:

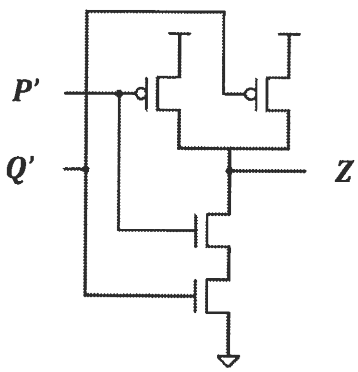
9:00 AM	[ ] AB1	
10:00 AM	[ ] AB2	
11:00 AM	[ ] AB3	
12:00 PM	[ ] AB4	
1:00 PM	[ ] AB5	[ ] ABA
2:00 PM	[ ] AB6	
3:00 PM	[ ] AB7	[ ] ABB
4:00 PM	[ ] AB8	[ ] ABC
5:00 PM	[ ] AB9	[ ] ABD

- **Be sure that your exam booklet has 8 pages.**
- **Write your name, netid and check discussion section on the title page.**
- **Do not tear the exam booklet apart.**
- **Use backs of pages for scratch work if needed.**
- **This is a closed book exam. You may not use a calculator.**
- **You are allowed one handwritten 8.5 x 11" sheet of notes (both sides).**
- **Absolutely no interaction between students is allowed.**
- **Clearly indicate any assumptions that you make.**
- **The questions are not weighted equally. Budget your time accordingly.**
- **Show your work.**

Problem 1	20 points	______
Problem 2	19 points	______
Problem 3	18 points	______
Problem 4	14 points	______
Problem 5	17 points	______
Problem 6	12 points	______
Total	100 points	______

Problem 1 (20 points): CMOS and Boolean properties

1. (5 points) Circle the correct choice for each statement. **The inputs are inverted.**



a. The output *Z* equals:

P* AND *Q ***P* OR *Q*** ***P* NOR *Q*** ***P* XOR *Q*** ***P* XNOR *Q***

b. This example best illustrates the Boolean property:

DeMorgan's **Absorption** **No-Name** **Consensus**

2. (15 points) Let *G*(*x*,*y*,*z*) and *H*(*x*,*y*,*z*) be the 3-variable functions whose K-maps are given below.

G(x,y,z)

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	0	0

H(x,y,z)

x \ yz	00	01	11	10
0	0	1	1	1
1	1	1	1	0

a. Express *H*(*x*,*y*,*z*) in **canonical POS form**

i. Using the **variables** *x*, *y*, *z*:

$H(x,y,z) =$ $(x+y+z)(x'+y'+z)$

ii. Using the **maxterm M_i notation**:

$H(x,y,z) =$ $M_0 M_6$

b. Using your expression for *H* from part a.i), give the exact **dual of H**:

dual of $H(x,y,z) =$ $xyz + x'y'z$

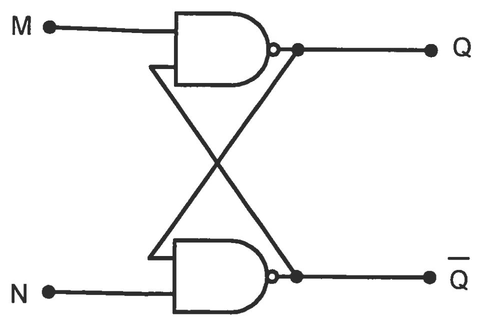
c. Complete the K-map (below) for function *F*, so that ***F* ⊕ *G* = *H*.** **You must use don't cares wherever possible.**

F(x,y,z)

x \ yz	00	01	11	10
0	0	X	X	1
1	1	X	1	0

Problem 2 (19 points): Sequential logic

1. (10 points) Consider the sequential feedback circuit shown below.



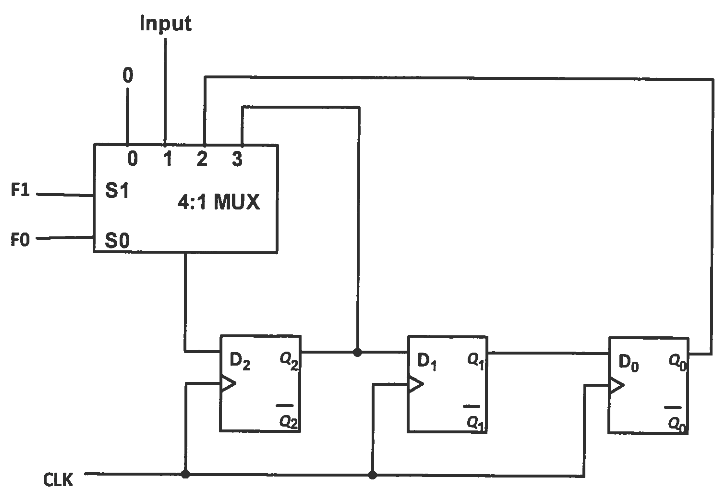
a. Complete the next-state table for this circuit

M	N	Q^+
0	0	Forbidden
0	1	1
1	0	0
1	1	Q

b. Express the next state Q^+ as a function of *M*, *N*, and *Q* in **SOP form**.

$Q^+ = M'N + MNQ = M' + MNQ = M'NQ' + M'NQ + MNQ$
$= M'N + NQ = M' + NQ$

2. (9 points) Consider a 3-bit shift register that has the following diagram:



a. Determine the functionality of the register by completing the following table

F_1	F_0	Operation
0	0	Unused
0	1	Logical shift right
1	0	Circular shift right
1	1	Arithmetic shift right

b. If the shift register initially stores $Q_2Q_1Q_0$=100 and *Input*=0, what is stored in the register after one clock pulse and

$F_1 F_0$ = 01? 010

$F_1 F_0$ = 10? 010 (Assume again that 100 is stored before the operation.)

$F_1 F_0$ = 11? 110 (Assume again that 100 is stored before the operation.)

Problem 3 (18 points)

Consider the 4-variable function *f(w,x,y,z)*, with the following K-map (drawn twice).

minimal SOP

f(w,x,y,z)

wx \ yz	00	01	11	10
00	1	x	0	1
01	0	1	1	0
11	0	0	1	x
10	1	1	0	1

minimal POS

f(w,x,y,z)

wx \ yz	00	01	11	10
00	1	x	0	1
01	0	1	1	0
11	0	0	1	x
10	1	1	0	1

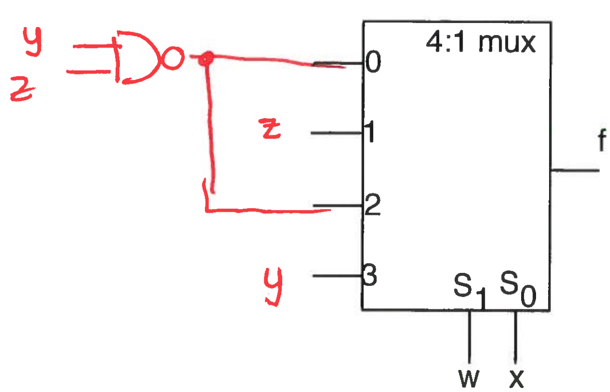
1. Give a **minimal SOP** expression for *f(w,x,y,z)* and show the corresponding loops on the **left map**.

   min SOP: $\bar{x}\bar{z} + \bar{x}\bar{y} + \{\bar{w}\bar{y}z + xyz$ or $\bar{w}xz + xyz$ or $\bar{w}xz + wxy\}$

2. Give a **minimal POS** expression for *f(w,x,y,z)* and show the corresponding loops on the **right map**.

   min POS: $(\bar{x}+z)(\bar{w}+\bar{x}+y)(x+\bar{y}+\bar{z})$

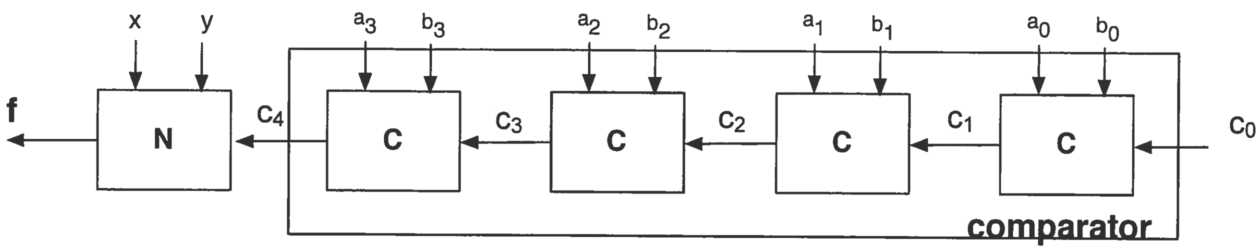
3. Implement *f* using **only a 4:1 multiplexer** (with select inputs $S_1S_0 = wx$) and **one NAND gate**. Complemented inputs are **not** available.



Problem 4 (14 points)

In this problem you will complete the design of the circuit shown below, which compares two 4-bit unsigned binary numbers $A=a_3a_2a_1a_0$ and $B=b_3b_2b_1b_0$ and outputs

$$f = \begin{cases} x \text{ NAND } y & \text{if } A < B \\ x' \text{ OR } y & \text{if } A \geq B \end{cases}$$



1. (8 points) Design cell C so that the comparator portion of the above circuit operates correctly and outputs

$$c_4 = \begin{cases} 0 & \text{if } A < B \\ 1 & \text{if } A \geq B \end{cases}$$

a. Specify the input c_0.

c_0 = 1

b. Express c_{i+1} in terms of c_i, a_i, b_i.

c_{i+1} = $a_i\bar{b}_i + a_ic_i + \bar{b}_ic_i$

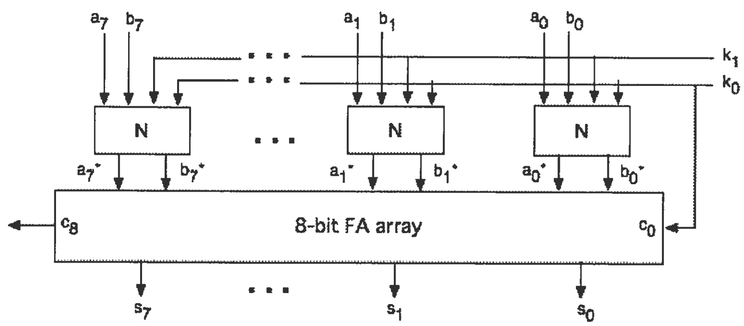
2. (6 points) Design the network N by giving a Boolean expression for f.

f = $\bar{c}_4(\overline{xy}) + c_4(\bar{x}+y)$

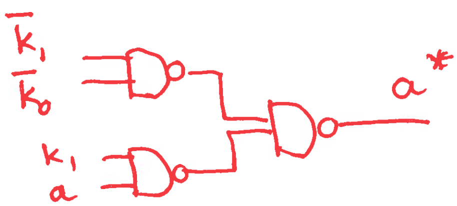
Problem 5 (17 points)

Shown below is an **8-bit arithmetic unit (AU)** which operates on two 8-bit **2's complement** numbers A and B. Each network N computes a* and b*, where:

$$a^* = k_1' \, k_0' + k_1 a$$
$$b^* = k_1 k_0' b' + k_0 b + k_1' k_0' a$$



1. (4 points) Give a **2-level NAND gate** implementation of a*. Assume complemented inputs are available.

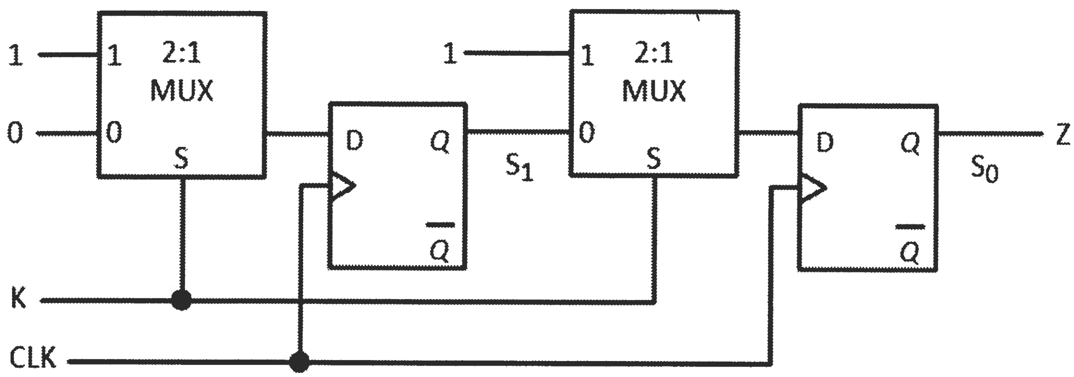


2. (13 points) Complete the table below.
   a. Give the values for a*, b*, c_0
   b. Specify the operation performed. Express your answer as an **arithmetic function (PLUS/MINUS)** of **A and B** (e.g., "a plus the complement of b" is not an appropriate response).

k_1	k_0	a*	b*	c_0	Operation performed as a function of A and B (e.g. A PLUS/MINUS B)
0	0	1	a	0	A MINUS 1
0	1	0	b	1	B PLUS 1
1	0	a	$\overline{b}$	0	A MINUS B MINUS 1
1	1	a	b	1	A PLUS B PLUS 1

Problem 6 (12 points): Finite State Machines

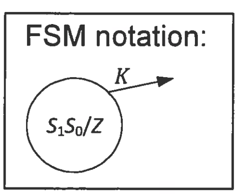
The circuit below is a 2-bit register that shifts right with serial input of 0 when K=0 and parallel loads with inputs of 1 when K=1.

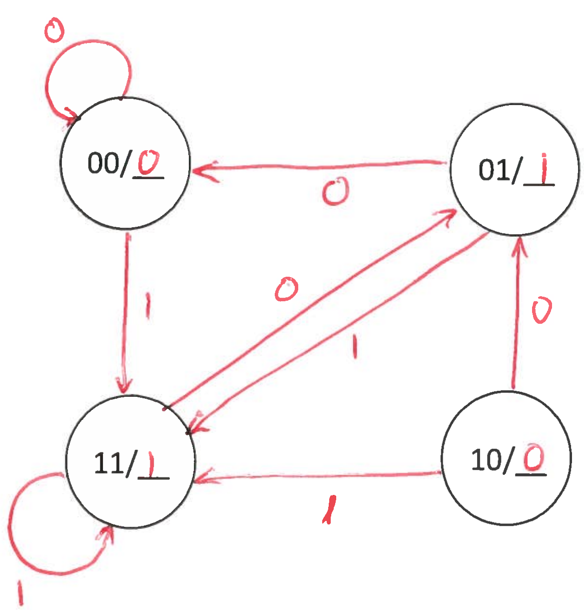


1. (6 points) Complete the state transition table for the circuit.

S_1	S_0	K	S_1^+	S_0^+	Z
0	0	0	0	0	0
0	0	1	1	1	0
0	1	0	0	0	1
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	0	1	1
1	1	1	1	1	1

2. (6 points) Complete the state transition diagram for the circuit.





Boolean algebra properties

Commutativity	$x \cdot y = y \cdot x$	$x + y = y + x$
Associativity	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$	$(x + y) + z = x + (y + z)$
Distributivity	$x \cdot (y + z) = x \cdot y + x \cdot z$	$x + y \cdot z = (x + y) \cdot (x + z)$
Idempotence	$x \cdot x = x$	$x + x = x$
Identity	$x \cdot 1 = x$	$x + 0 = x$
Null	$x \cdot 0 = 0$	$x + 1 = 1$
Complementarity	$x \cdot x' = 0$	$x + x' = 1$
Involution	$(x')' = x$	
DeMorgan's	$(x \cdot y)' = x' + y'$	$(x + y)' = x' \cdot y'$
Absorption	$x \cdot (x + y) = x$	$x + x \cdot y = x$
No-Name	$x \cdot (x' + y) = x \cdot y$	$x + x' \cdot y = x + y$
Consensus	$(x+y) \cdot (y+z) \cdot (x'+z) = (x+y) \cdot (x'+z)$	$x \cdot y + y \cdot z + x' \cdot z = x \cdot y + x' \cdot z$

Feel free to tear this page off and use it as scratch paper.