

**ECE 120 Second Midterm Exam
Fall 2016**

Tuesday, October 18, 2016

Name:	SOLUTIONS	NetID:	
Discussion Section:			
9:00 AM			
10:00 AM			
11:00 AM	<input type="checkbox"/> AB1	<input type="checkbox"/> AB8	
12:00 PM	<input type="checkbox"/> AB2	<input type="checkbox"/> AB9	
1:00 PM	<input type="checkbox"/> AB3	<input type="checkbox"/> ABA	
2:00 PM	<input type="checkbox"/> AB4	<input type="checkbox"/> ABB	
3:00 PM	<input type="checkbox"/> AB5		
4:00 PM	<input type="checkbox"/> AB6	<input type="checkbox"/> ABC	
5:00 PM	<input type="checkbox"/> AB7	<input type="checkbox"/> ABD	

- Be sure that your exam booklet has 11 pages.
- Write your name, netid and check discussion section on the title page.
- Do not tear the exam booklet apart, except for the last two pages.
- Use backs of pages for scratch work if needed.
- This is a closed book exam. You may not use a calculator.
- You are allowed one handwritten 8.5 x 11" sheet of notes (both sides).
- Absolutely no interaction between students is allowed.
- Clearly indicate any assumptions that you make.
- The questions are not weighted equally. Budget your time accordingly.
- Show your work.

Problem 1	14 points	_____
Problem 2	14 points	_____
Problem 3	14 points	_____
Problem 4	14 points	_____
Problem 5	14 points	_____
Problem 6	30 points	_____

Total	100 points	_____
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Problem 1 (14 points): K-maps and Don't Cares

Consider the 4-variable function $f(a,b,c,d)$, with the following K-map (drawn twice). **Note:** there are extra copies of this K-map on the last page of the exam. Use them for scratch work, but they will NOT be graded. Make sure you mark the two K-maps below correctly.

Minimal SOP					Minimal POS						
$f(a,b,c,d)$		cd				$f(a,b,c,d)$		cd			
		00	01	11	10			00	01	11	10
ab	00	1	0	0	1	ab	00	1	0	0	1
	01	X	0	0	0		01	X	0	0	0
	11	0	X	X	0		11	0	X	X	0
	10	X	1	1	X		10	X	1	1	X

1. (4 points) Give a **minimal SOP** expression for $f(a,b,c,d)$ and show the corresponding loops on the **left map**.

Minimal SOP: $f(a,b,c,d) = \underline{b'd' + dd}$

2. (3 points) Is the solution to part 1 unique? If it is not, write a different minimal SOP solution.

Circle one: **UNIQUE**

NOT UNIQUE

If not unique, write a different minimal SOP solution (but do not mark the loops):

Minimal SOP: $f(a,b,c,d) = \underline{b'd' + ab'}$

3. (4 points) Give a **minimal POS** expression for $f(a,b,c,d)$ and show the corresponding loops on the **right map**.

Minimal POS: $f(a,b,c,d) = \underline{b'(a+d')}$

4. (3 points) Is the solution to part 3 unique? If it is not, write a different minimal POS solution.

Circle one: **UNIQUE**

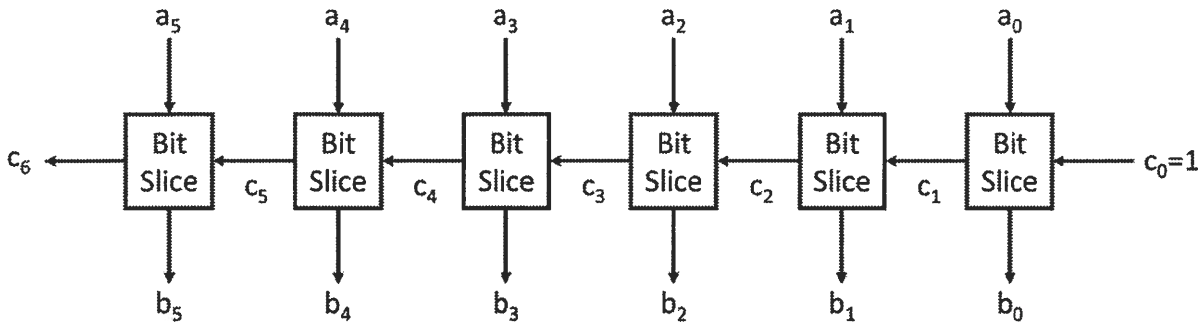
NOT UNIQUE

If not unique, write a different minimal POS solution (but do not mark the loops):

Minimal POS: $f(a,b,c,d) = \underline{\hspace{10em}}$

Problem 2 (14 points): Bit-sliced Design

The circuit shown below is intended to negate a 6-bit 2's complement number. For example, if the input $a_5a_4a_3a_2a_1a_0 = 110100$, then the output $b_5b_4b_3b_2b_1b_0 = 001100$. Note that the initial carry-in bit c_0 is set to 1 in this design.



1. (8 points) Each bit slice is identical. Complete the truth table for a single bit slice and write the minimal SOP expressions.

c_i	a_i	c_{i+1}	b_i
0	0	0	1
0	1	0	0
1	0	1	0
1	1	0	1

Minimal SOP: $c_{i+1} = c_i a_i'$

Minimal SOP: $b_i = c_i' a_i' + c_i a_i$

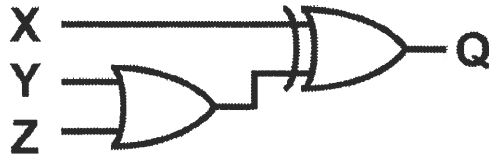
2. (6 points) Assume that the b_i and c_{i+1} outputs of each bit slice are implemented using 2-level AND-OR networks based on their minimal SOP expressions. In this question, count AND and OR gates, but do not count NOT gates.

How many gates are required in the entire circuit (all bit slices)? 24

How many gates are there along one of the longest paths from a_0 to b_5 ? 7

Problem 3 (14 points): Sequential Feedback Analysis

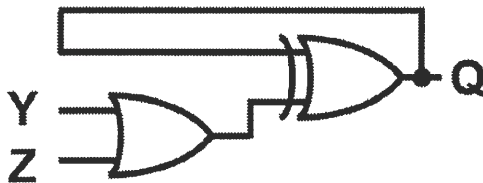
1. (8 points) Complete the truth table for the circuit shown below made out of one OR gate and one XOR gate.



X	Y	Z	Q
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1

X	Y	Z	Q
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

2. (6 points) For the circuit shown below, which combinations of inputs YZ cause the output Q to oscillate between 0 and 1? In other words, which values of YZ prevent the output Q from holding a stable value?



Circle all oscillating input combinations YZ:

00

01

10

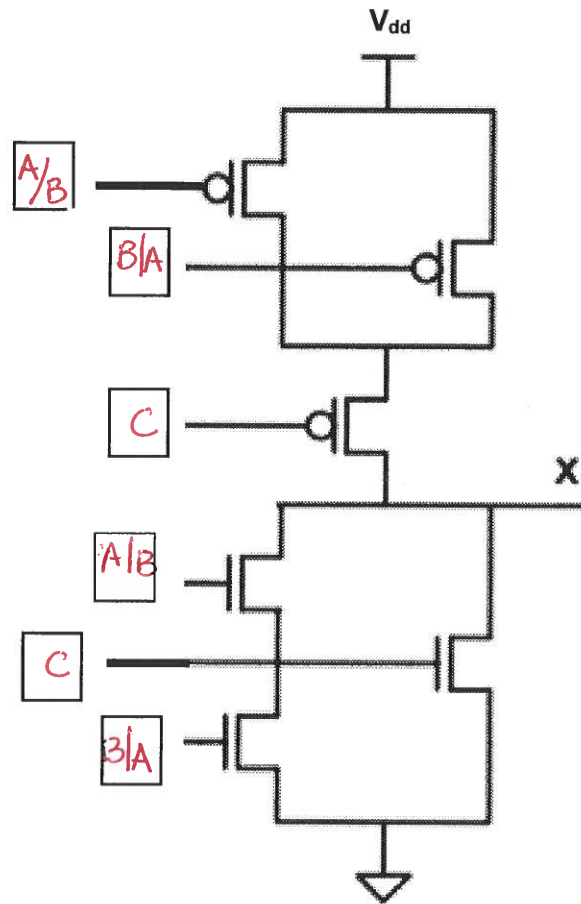
11

Problem 4 (14 points): CMOS Gates, Boolean Expressions and 2-Level Design

1. (6 points) The transistor-level circuit below implements the logic equation:

$$X = \overline{AB} + C$$

Label the inputs to all transistors.

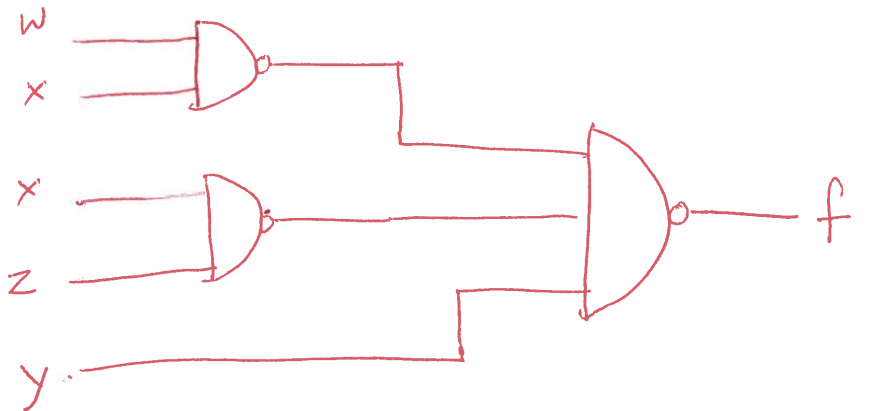


Problem 4 (14 points), continued:

2. (4 points) As a consultant to the company Dontel Corp., your task is to design a system that counts the number of 1s present in a 3-bit pattern ABC to output a two-bit number $X_1 X_0$ (representing that count in binary). Describe the behavior of your system as a truth table.

A	B	C	X_1	X_0
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

3. (4 points) Implement $f=wx+y'+xz$ as a 2-level NAND-NAND circuit using the minimum number of gates. *May use complemented inputs available.*



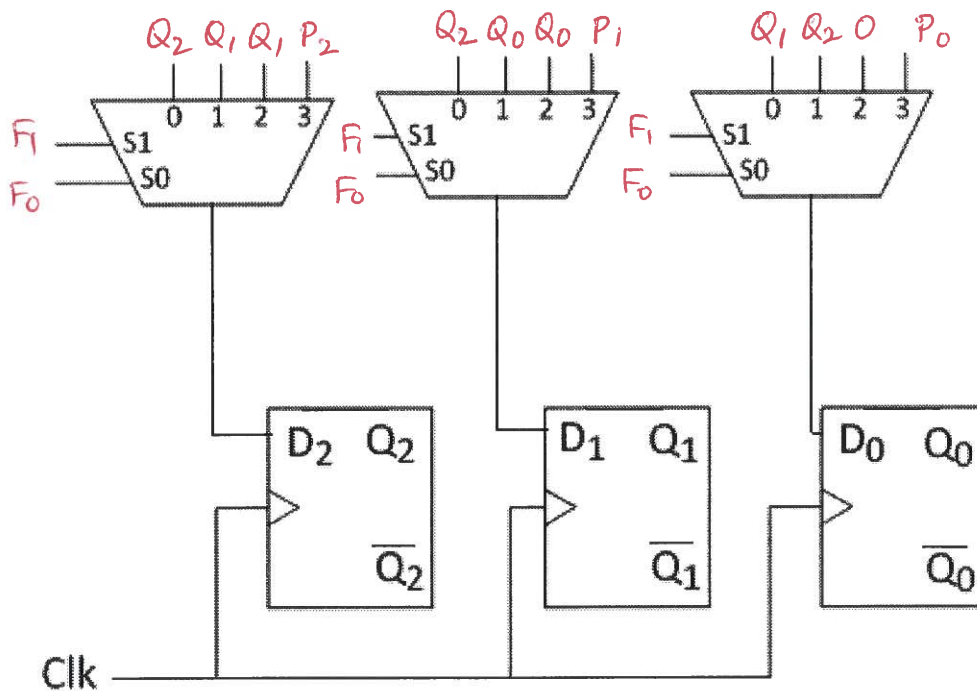
Problem 5 (14 points): Basic Registers

Consider a 3-bit shift register that has the functionality specified in the table to the right.

1. (10 points) Label the inputs to the muxes to complete the design of this 3-bit register that performs the operations listed in the table. Parallel load input i is labeled as P_i . Use labels and do not draw any additional gates or wires.

F_1	F_0	Operation
0	0	Arithmetic shift right
0	1	Circular shift left
1	0	Arithmetic shift left
1	1	Parallel load

Assume 2's Complement numbers.



2. (2 points) If the shift register initially stores 110, what is stored in the register after one clock cycle when $F_1F_0 = 00$?

Answer: 111

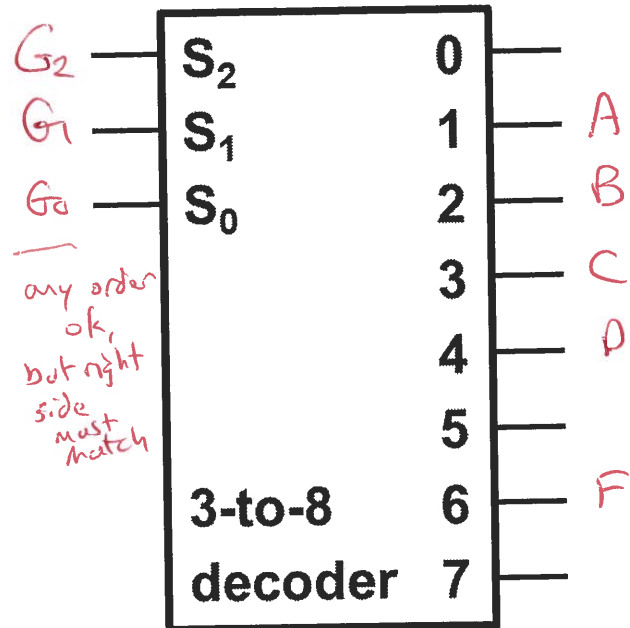
3. (2 points) Assume again that 110 is stored in the register. What is stored in the register after one clock cycle when $F_1F_0 = 10$?

Answer: 100

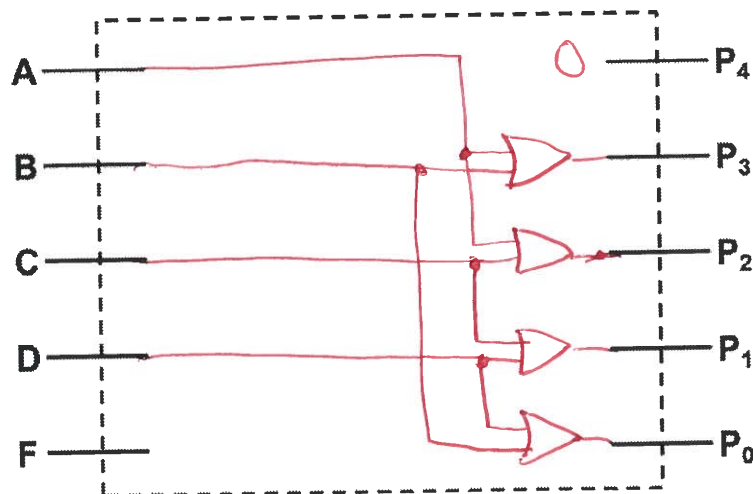
Problem 6 (30 points): Designing Logic with Components

Prof. Lumetta needs your help again. He is still fixated on calculating grades. **Parts 1 through 4 do not depend on one another.**

1. (8 points) Help Prof. Lumetta use a decoder to translate the letter grade (A, B, C, D, or F) into a baseline GPA value. The typed ASCII letter is given as 7-bit input $G=G_6G_5G_4G_3G_2G_1G_0$. The letter 'A' has ASCII code 0x41, 'B' is 0x42, and so forth. **Label decoder inputs and outputs as necessary on the 3-to-8 decoder to the right to produce signals for the grades A, B, C, D, and F based on bits of input G. Leave any unused decoder outputs unlabeled.**



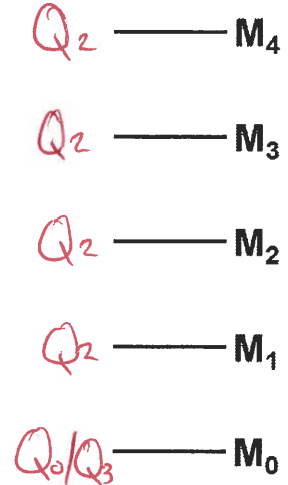
2. (8 points) Now use the grade signals A, B, C, D, and F to compute a 5-bit 2's complement GPA value P. The grade A has decimal value 12, B has value 9, C has value 6, D has value 3, and F has value 0. You may use **up to five gates** of any type (AND, OR, NOT, XOR, NAND, NOR, or XNOR) as well as **logic values 0 and 1**.



can also use XOR
can also write with NORs,
such as $P_0 = (A+C+F)'$

Problem 6 (30 points), continued:

3. (6 points) Grades can be modified by a + or a -. For example, B+ or A-. Process a second 7-bit ASCII character $Q=Q_6Q_5Q_4Q_3Q_2Q_1Q_0$, which is always one of the following: '+' (ASCII code 0x2B), '-' (0x2D), or SPACE (0x20). Design a circuit to produce a 5-bit 2's complement GPA modifier M . M should have a decimal value of 1 when Q is '+', a value of -1 when Q is '-', and a value of 0 when Q is SPACE. Design your circuit to the right using only labels (no components, and no gates). In other words, **label each output bit with an appropriate input variable**.



4. (8 points) Prof. Lumetta stores a sum of GPA values in a 5-bit register, R . Using the values that you have computed, P and M , and the current sum R , you must now design a circuit to produce a new sum, T . **Use only combinational logic components** that you have seen in class, such as adders, comparators, muxes, and decoders. Draw your circuit in the box below, **labeling all inputs, outputs, and components clearly, including the number of bits in each signal**. Do not worry about overflow for this problem.

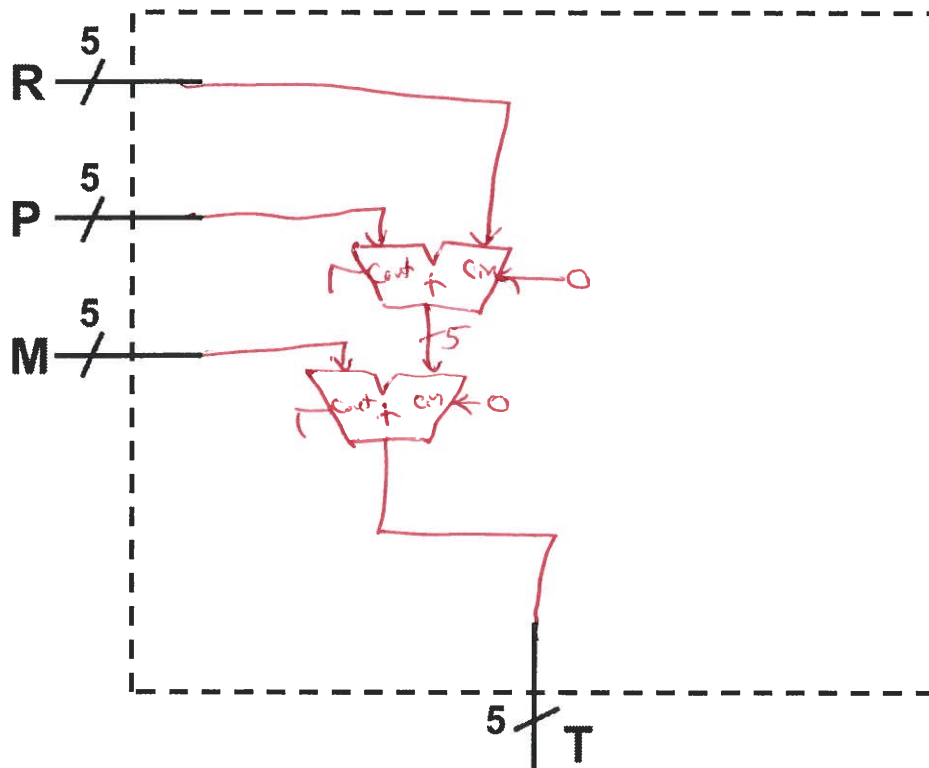


Table of ASCII Characters

Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex
(nul)	0	00	(sp)	32	20	@	64	40	`	96	60
(soh)	1	01	!	33	21	A	65	41	a	97	61
(stx)	2	02	"	34	22	B	66	42	b	98	62
(etx)	3	03	#	35	23	C	67	43	c	99	63
(eot)	4	04	\$	36	24	D	68	44	d	100	64
(enq)	5	05	%	37	25	E	69	45	e	101	65
(ack)	6	06	&	38	26	F	70	46	f	102	66
(bel)	7	07	'	39	27	G	71	47	g	103	67
(bs)	8	08	(40	28	H	72	48	h	104	68
(ht)	9	09)	41	29	I	73	49	i	105	69
(lf)	10	0a	*	42	2a	J	74	4a	j	106	6a
(vt)	11	0b	+	43	2b	K	75	4b	k	107	6b
(ff)	12	0c	,	44	2c	L	76	4c	l	108	6c
(cr)	13	0d	-	45	2d	M	77	4d	m	109	6d
(so)	14	0e	.	46	2e	N	78	4e	n	110	6e
(si)	15	0f	/	47	2f	O	79	4f	o	111	6f
(dle)	16	10	0	48	30	P	80	50	p	112	70
(dc1)	17	11	1	49	31	Q	81	51	q	113	71
(dc2)	18	12	2	50	32	R	82	52	r	114	72
(dc3)	19	13	3	51	33	S	83	53	s	115	73
(dc4)	20	14	4	52	34	T	84	54	t	116	74
(nak)	21	15	5	53	35	U	85	55	u	117	75
(syn)	22	16	6	54	36	V	86	56	v	118	76
(etb)	23	17	7	55	37	W	87	57	w	119	77
(can)	24	18	8	56	38	X	88	58	x	120	78
(em)	25	19	9	57	39	Y	89	59	y	121	79
(sub)	26	1a	:	58	3a	Z	90	5a	z	122	7a
(esc)	27	1b	;	59	3b	[91	5b	{	123	7b
(fs)	28	1c	<	60	3c	\	92	5c		124	7c
(gs)	29	1d	=	61	3d]	93	5d	}	125	7d
(rs)	30	1e	>	62	3e	^	94	5e	~	126	7e
(us)	31	1f	?	63	3f	_	95	5f	(del)	127	7f

Boolean algebra properties

Commutativity	$x \cdot y = y \cdot x$	$x + y = y + x$
Associativity	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$	$(x + y) + z = x + (y + z)$
Distributivity	$x \cdot (y + z) = x \cdot y + x \cdot z$	$x + y \cdot z = (x + y) \cdot (x + z)$
Idempotence	$x \cdot x = x$	$x + x = x$
Identity	$x \cdot 1 = x$	$x + 0 = x$
Null	$x \cdot 0 = 0$	$x + 1 = 1$
Complementarity	$x \cdot x' = 0$	$x + x' = 1$
Involution		$(x')' = x$
DeMorgan's	$(x \cdot y)' = x' + y'$	$(x + y)' = x' \cdot y'$
Absorption	$x \cdot (x + y) = x$	$x + x \cdot y = x$
No-Name	$x \cdot (x' + y) = x \cdot y$	$x + x' \cdot y = x + y$
Consensus	$(x + y) \cdot (y + z) \cdot (x' + z) = (x + y) \cdot (x' + z)$	$x \cdot y + y \cdot z + x' \cdot z = x \cdot y + x' \cdot z$

Feel free to tear this page off and use it as scratch paper.

Extra copies of K-map for problem 1 (use as scratch copies, we will NOT grade them)

Minimal SOP

$f(a,b,c,d)$		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal POS

$f(a,b,c,d)$		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal SOP

$f(a,b,c,d)$		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal POS

$f(a,b,c,d)$		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal SOP

$f(a,b,c,d)$		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal POS

$f(a,b,c,d)$		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X