ECE 120 Second Midterm Exam
Fall 2016

Tuesday, October 18, 2016

Name: ______________________ NetID: ______________

Discussion Section:

9:00 AM		
10:00 AM		
11:00 AM	[ ] AB1	[ ] AB8
12:00 PM	[ ] AB2	[ ] AB9
1:00 PM	[ ] AB3	[ ] ABA
2:00 PM	[ ] AB4	[ ] ABB
3:00 PM	[ ] AB5	
4:00 PM	[ ] AB6	[ ] ABC
5:00 PM	[ ] AB7	[ ] ABD

- **Be sure that your exam booklet has 11 pages.**
- **Write your name, netid and check discussion section on the title page.**
- **Do not tear the exam booklet apart, except for the last two pages.**
- **Use backs of pages for scratch work if needed.**
- **This is a closed book exam. You may not use a calculator.**
- **You are allowed one handwritten 8.5 x 11" sheet of notes (both sides).**
- **Absolutely no interaction between students is allowed.**
- **Clearly indicate any assumptions that you make.**
- **The questions are not weighted equally. Budget your time accordingly.**
- **Show your work.**

Problem 1	14 points	___________
Problem 2	14 points	___________
Problem 3	14 points	___________
Problem 4	14 points	___________
Problem 5	14 points	___________
Problem 6	30 points	___________
Total	100 points	___________

Problem 1 (14 points): K-maps and Don't Cares

Consider the 4-variable function *f*(*a,b,c,d*), with the following K-map (drawn twice). **Note:** there are extra copies of this K-map on the last page of the exam. Use them for scratch work, but they will NOT be graded. Make sure you mark the two K-maps below correctly.

Minimal SOP

f(a,b,c,d) cd

ab	00	01	11	10
00	1	0	0	1
01	X	0	0	0
11	0	X	X	0
10	X	1	1	X

Minimal POS

f(a,b,c,d) cd

ab	00	01	11	10
00	1	0	0	1
01	X	0	0	0
11	0	X	X	0
10	X	1	1	X

1. **(4 points)** Give a **minimal SOP** expression for *f*(*a,b,c,d*) and show the corresponding loops on the **left map**.

   Minimal SOP: **f(a,b,c,d)=**____

2. **(3 points)** Is the solution to part 1 unique? If it is not, write a different minimal SOP solution.

   Circle one: **UNIQUE** **NOT UNIQUE**

   If not unique, write a different minimal SOP solution (but do not mark the loops):

   Minimal SOP: **f(a,b,c,d)=**____

3. **(4 points)** Give a **minimal POS** expression for *f*(*a,b,c,d*) and show the corresponding loops on the **right map**.

   Minimal POS: **f(a,b,c,d)=**____

4. **(3 points)** Is the solution to part 3 unique? If it is not, write a different minimal POS solution.

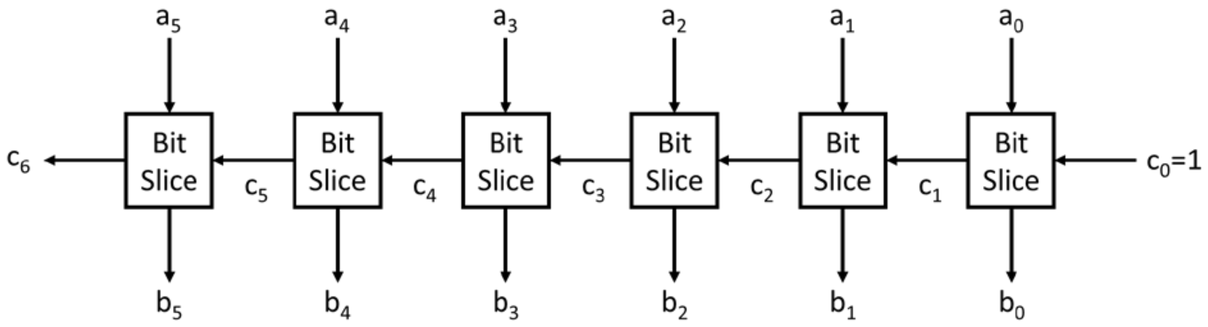
   Circle one: **UNIQUE** **NOT UNIQUE**

   If not unique, write a different minimal POS solution (but do not mark the loops):

   Minimal SOP: **f(a,b,c,d)=**____

Problem 2 (14 points): Bit-sliced Design

The circuit shown below is intended to negate a 6-bit 2's complement number. For example, if the input $a_5a_4a_3a_2a_1a_0$ = 110100, then the output $b_5b_4b_3b_2b_1b_0$ = 001100. Note that the initial carry-in bit c_0 is set to 1 in this design.



1. **(8 points)** Each bit slice is identical. Complete the truth table for a single bit slice and write the minimal SOP expressions.

c_i	a_i	c_{i+1}	b_i
0	0		
0	1		
1	0		
1	1		

Minimal SOP: **c_{i+1}=**____

Minimal SOP: **b_i=**____

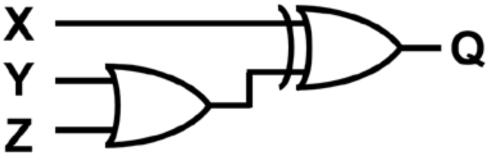
2. **(6 points)** Assume that the b_i and c_{i+1} outputs of each bit slice are implemented using 2-level AND-OR networks based on their minimal SOP expressions. **In this question, count AND and OR gates, but do not count NOT gates.**

   How many gates are required in the entire circuit (all bit slices)? ________________________

   How many gates are there along one of the longest paths from a_0 to b_5? __________________

Problem 3 (14 points): Sequential Feedback Analysis

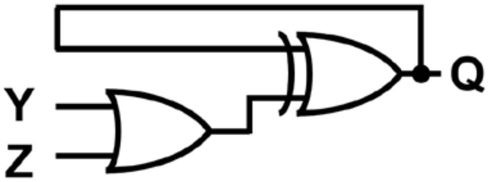
1. **(8 points)** Complete the truth table for the circuit shown below made out of one OR gate and one XOR gate.



X	Y	Z	Q
0	0	0	
0	0	1	
0	1	0	
0	1	1	

X	Y	Z	Q
1	0	0	
1	0	1	
1	1	0	
1	1	1	

2. **(6 points)** For the circuit shown below, which combinations of inputs YZ cause the output Q to oscillate between 0 and 1? In other words, which values of YZ prevent the output Q from holding a stable value?



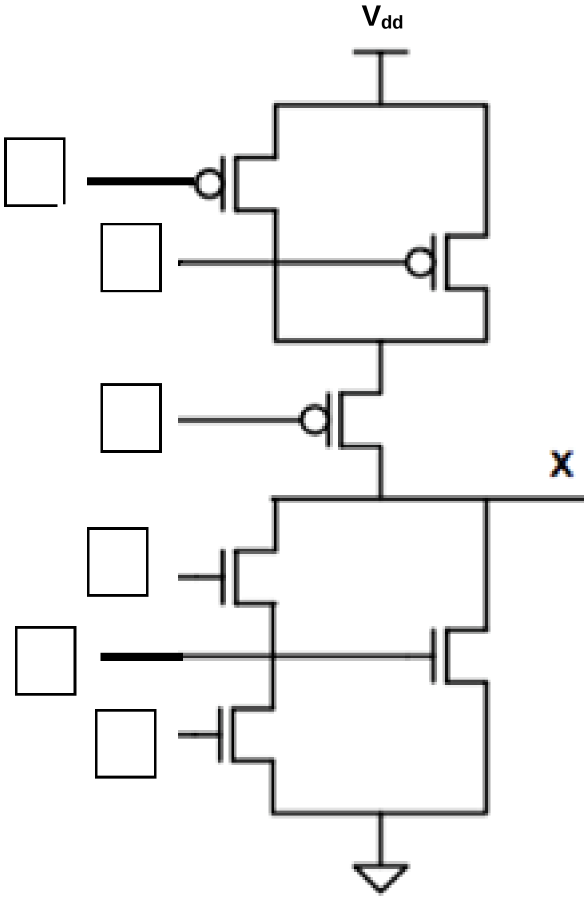
Circle all oscillating input combinations YZ: 00 01 10 11

Problem 4 (14 points): CMOS Gates, Boolean Expressions and 2-Level Design

1. **(6 points)** The transistor-level circuit below implements the logic equation:

$$X = \overline{AB + C}$$

Label the inputs to all transistors.



Problem 4 (14 points), continued:

2. **(4 points)** As a consultant to the company Dontel Corp., your task is to design a system that counts the number of 1s present in a 3-bit pattern ABC to output a two-bit number X_1 X_0 (representing that count in binary). Describe the behavior of your system as a truth table.

A	**B**	**C**	**X_1**	**X_0**
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

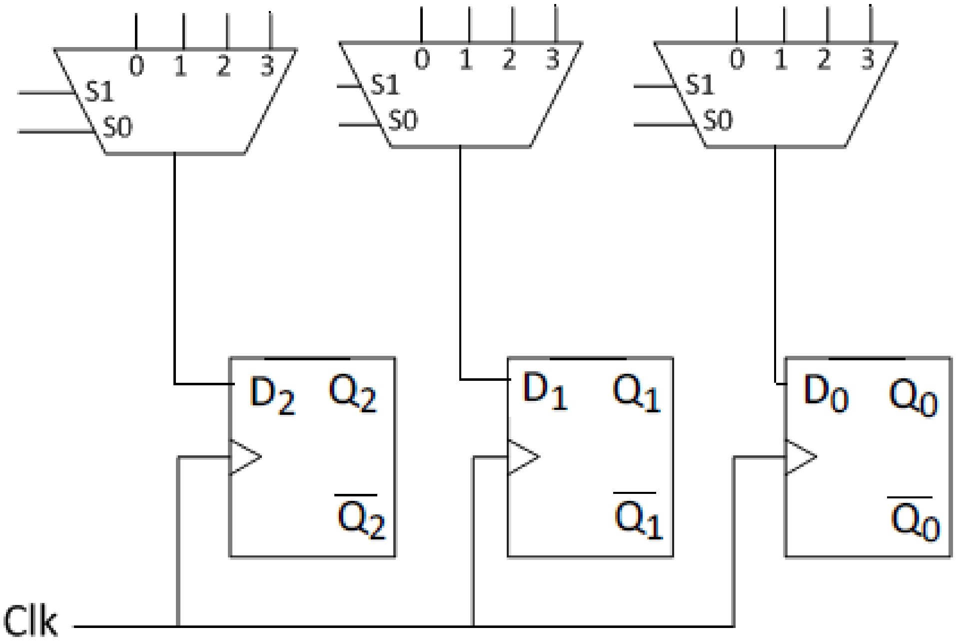
3. **(4 points)** Implement **f=wx+y'+xz** as a 2-level NAND-NAND circuit using the minimum number of gates.

Problem 5 (14 points): Basic Registers

Consider a 3-bit shift register that has the functionality specified in the table to the right.

1. **(10 points)** Label the inputs to the muxes to complete the design of this 3-bit register that performs the operations listed in the table. Parallel load input *i* is labeled as P_i. **Use labels and do not draw any additional gates or wires**.

F_1	F_0	Operation
0	0	Arithmetic shift right
0	1	Circular shift left
1	0	Arithmetic shift left
1	1	Parallel load



2. **(2 points)** If the shift register initially stores 110, what is stored in the register after one clock cycle when $F_1F_0 = 00$?

   Answer: ______________________________________

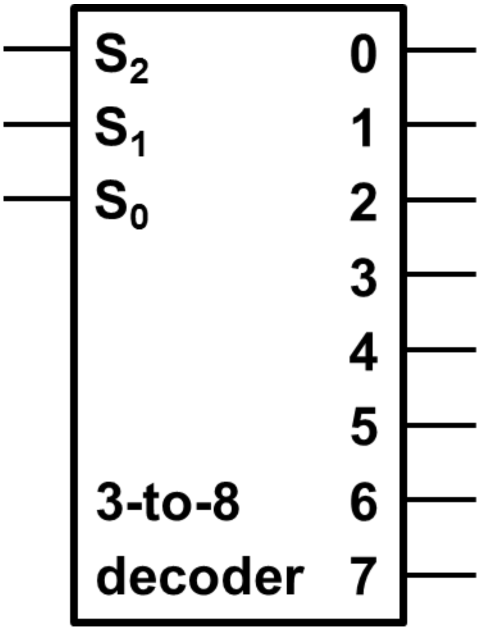
3. **(2 points)** Assume again that 110 is stored in the register. What is stored in the register after one clock cycle when $F_1F_0 = 10$?

   Answer: ______________________________________

Problem 6 (30 points): Designing Logic with Components

Prof. Lumetta needs your help again. He is still fixated on calculating grades.
Parts 1 through 4 do not depend on one another.

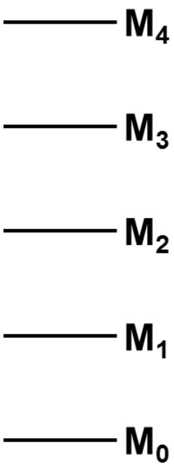
1. **(8 points)** Help Prof. Lumetta use a decoder to translate the letter grade (A, B, C, D, or F) into a baseline GPA value. The typed ASCII letter is given as 7-bit input $G=G_6G_5G_4G_3G_2G_1G_0$. The letter ′A′ has ASCII code 0x41, ′ B′ is 0x42, and so forth. **Label decoder inputs and outputs as necessary on the 3-to-8 decoder to the right** to produce signals for the grades A, B, C, D, and F based on bits of input G. Leave any unused decoder outputs unlabeled.



2. **(8 points)** Now use the grade signals A, B, C, D, and F to compute a 5-bit 2’s complement GPA value P. The grade A has decimal value 12, B has value 9, C has value 6, D has value 3, and F has value 0. You may use **up to five gates** of any type (AND, OR, NOT, XOR, NAND, NOR, or XNOR) as well as **logic values 0 and 1**.

Problem 6 (30 points), continued:

3. **(6 points)** Grades can be modified by a + or a -. For example, B+ or A-. Process a second 7-bit ASCII character $Q=Q_6Q_5Q_4Q_3Q_2Q_1Q_0$, which is always one of the following: ′+′ (ASCII code 0x2B), ′ -′ (0x2D), or SPACE (0x20). Design a circuit to produce a 5-bit 2’s complement GPA modifier M. M should have a decimal value of 1 when Q is ′ +′, a value of -1 when Q is ′ -′, and a value of 0 when Q is SPACE. Design your circuit to the right using only labels (no components, and no gates). In other words, **label each output bit with an appropriate input variable**.



4. **(8 points)** Prof. Lumetta stores a sum of GPA values in a 5-bit register, R. Using the values that you have computed, P and M, and the current sum R, you must now design a circuit to produce a new sum, T. **Use only combinational logic components** that you have seen in class, such as adders, comparators, muxes, and decoders. Draw your circuit in the box below, **labeling all inputs, outputs, and components clearly, including the number of bits in each signal**. **Do not worry about overflow for this problem.**

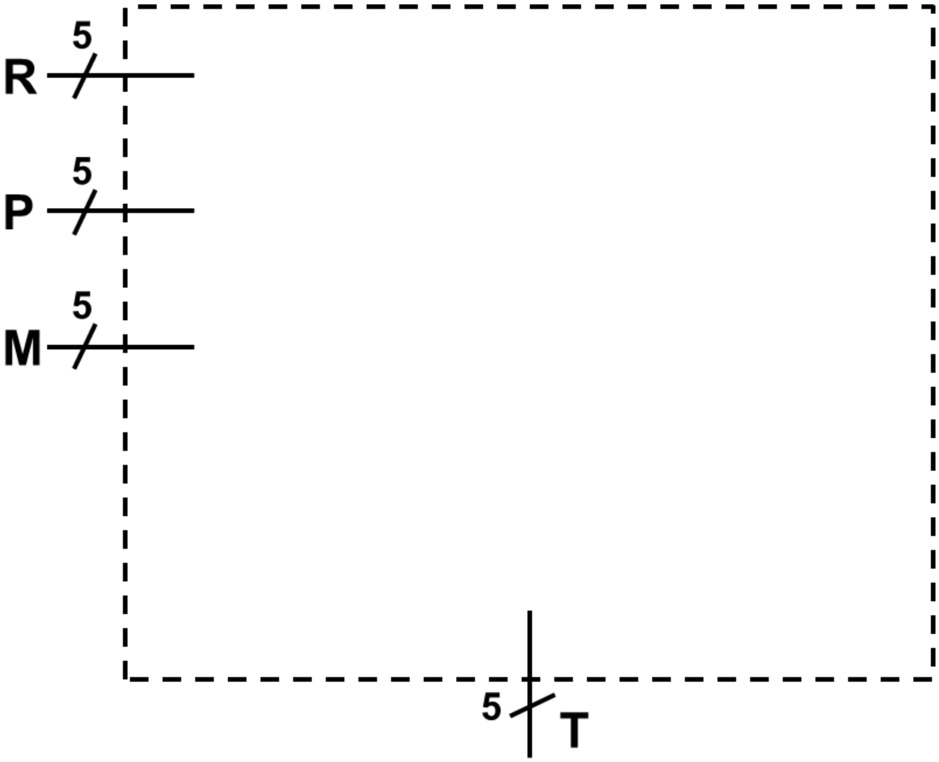


Table of ASCII Characters

Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex
(nul)	0	00	(sp)	32	20	@	64	40	`	96	60
(soh)	1	01	!	33	21	A	65	41	a	97	61
(stx)	2	02	"	34	22	B	66	42	b	98	62
(etx)	3	03	#	35	23	C	67	43	c	99	63
(eot)	4	04	$	36	24	D	68	44	d	100	64
(enq)	5	05	%	37	25	E	69	45	e	101	65
(ack)	6	06	&	38	26	F	70	46	f	102	66
(bel)	7	07	'	39	27	G	71	47	g	103	67
(bs)	8	08	(	40	28	H	72	48	h	104	68
(ht)	9	09	)	41	29	I	73	49	i	105	69
(lf)	10	0a	*	42	2a	J	74	4a	j	106	6a
(vt)	11	0b	+	43	2b	K	75	4b	k	107	6b
(ff)	12	0c	,	44	2c	L	76	4c	l	108	6c
(cr)	13	0d	-	45	2d	M	77	4d	m	109	6d
(so)	14	0e	.	46	2e	N	78	4e	n	110	6e
(si)	15	0f	/	47	2f	O	79	4f	o	111	6f
(dle)	16	10	0	48	30	P	80	50	p	112	70
(dc1)	17	11	1	49	31	Q	81	51	q	113	71
(dc2)	18	12	2	50	32	R	82	52	r	114	72
(dc3)	19	13	3	51	33	S	83	53	s	115	73
(dc4)	20	14	4	52	34	T	84	54	t	116	74
(nak)	21	15	5	53	35	U	85	55	u	117	75
(syn)	22	16	6	54	36	V	86	56	v	118	76
(etb)	23	17	7	55	37	W	87	57	w	119	77
(can)	24	18	8	56	38	X	88	58	x	120	78
(em)	25	19	9	57	39	Y	89	59	y	121	79
(sub)	26	1a	:	58	3a	Z	90	5a	z	122	7a
(esc)	27	1b	;	59	3b	[	91	5b	{	123	7b
(fs)	28	1c	<	60	3c	\	92	5c	|	124	7c
(gs)	29	1d	=	61	3d	]	93	5d	}	125	7d
(rs)	30	1e	>	62	3e	^	94	5e	~	126	7e
(us)	31	1f	?	63	3f	_	95	5f	(del)	127	7f

Boolean algebra properties

Commutativity	x·y = y·x	x + y = y + x
Associativity	(x·y)·z = x·(y·z)	(x + y) + z = x + (y + z)
Distributivity	x·(y + z) = x·y + x·z	x + y·z = (x + y)·(x + z)
Idempotence	x·x = x	x + x = x
Identity	x·1 = x	x + 0 = x
Null	x·0 = 0	x + 1 = 1
Complementarity	x·x’ = 0	x + x’ = 1
Involution	(x’)’ = x	
DeMorgan’s	(x·y)’ = x’ + y’	(x + y)’ = x’·y’
Absorption	x·(x + y) = x	x + x·y = x
No-Name	x·(x’ + y) = x·y	x + x’·y = x + y
Consensus	(x+y)·(y+z)·(x’+z) = (x+y)·(x’+z)	x·y + y·z + x’·z = x·y + x’·z

Feel free to tear this page off and use it as scratch paper.

Extra copies of K-map for problem 1 (use as scratch copies, we will NOT grade them)

Minimal SOP

f(a,b,c,d)		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal POS

f(a,b,c,d)		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal SOP

f(a,b,c,d)		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal POS

f(a,b,c,d)		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal SOP

f(a,b,c,d)		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X

Minimal POS

f(a,b,c,d)		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	X	0	0	0
	11	0	X	X	0
	10	X	1	1	X