# ECE199JL Final Exam, Fall 2012 Tuesday 18 December 

| Name and UIUC Net ID: |
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- Be sure that your exam booklet has 13 pages.
- Write your name at the top of each page.
- This is a closed book exam.
- We have included a scratch sheet and two LC-3 reference pages.
- Appendix $A$ of the textbook is available to you on request.
- You are allowed FOUR $8.5 \times 11$ " sheets of notes.
- Absolutely no interaction between students is allowed.
- Show all of your work.
- Challenge questions are marked with ***.
- Don't panic, and good luck!
"I think there is a world market for maybe five computers."
—Thomas Watson (Chairman of IBM), 1943

| Problem 1 | 10 points |
| :---: | :---: |
| Problem 2 | 15 points |
| Problem 3 | 15 points |
| Problem 4 | 15 points |
| Problem 5 | 25 points |
| Problem 6 | 10 points |
| Problem 7 | 10 points |
| Total | 100 points |

## Problem 1 (10 points): Representations

Part A (3 points): Explain why an $N$-bit signed magnitude representation allows you to represent only $2^{N}-1$ different numbers.

Part B (4 points): Two $N$-bit 2's complement numbers, $A$ and $B$, are added to find their sum $S$, as shown to the right.
Write a Boolean expression for the overflow condition for the addition in terms of the variables shown.

$$
\begin{array}{r}
A_{N-1} A_{N-2} \ldots A_{2} A_{1} A_{0} \\
+\quad B_{N-1} B_{N-2} \ldots B_{2} B_{1} B_{0} \\
\hline S_{N-1} S_{N-2} \ldots S_{2} S_{1} S_{0}
\end{array}
$$

Part C (3 points): As you know, addition of two IEEE single-precision floating-point numbers is not associative. In other words, for some values of $A, B$, and $C$,

$$
(A+B)+C \neq A+(B+C)
$$

Give an example of values for $A, B$, and $C$ for which this lack of associativity holds (write decimal numbers or scientific notation-you need not translate to the binary representation for IEEE floating-point!).

A $\qquad$

B $\qquad$
C $\qquad$

## Problem 2 (15 points): Logic

The block diagram below illustrates a specialized $N$-bit unsigned comparator. The comparator operates on two unsigned numbers, $A$ and $B$, to produce outputs $P_{N}$ and $Q_{N}$ with meanings defined in the table to the right.

| $P_{N}$ | $Q_{N}$ | Meaning |
| :---: | :---: | :--- |
| 0 | 0 | $A<B$ and both $A$ and $B$ are odd |
| 0 | 1 | $A \geq B$ and both $A$ and $B$ are odd |
| 1 | 0 | $A<B$ and $(A$ and $B$ are not both odd) |
| 1 | 1 | $A \geq B$ and $(A$ and $B$ are not both odd) |



Part A (5 points): Design the even/odd detector. Show all work, including drawing a gate-level diagram implementing outputs $P$ and $Q$ in terms of inputs $A$ and $B$.

Part B (10 points): Design the general bit slice for this comparator. Show all work, including drawing a gate-level diagram implementing outputs $P$ and $Q$ in terms of inputs $A, B, R$, and $S$.

## Problem 3 (15 points): Finite State Machines

Part A (5 points): Professor Lumetta promised Professor Cangellaris to design a holiday light display for the new ECE building, but Lumetta has been too busy writing exam problems!

The design to the right shows what he needs: combinational logic that translates the output of a binary counter (counts upward) into RGB signals according to the following repeating sequence in the table below.


The $R S T$ input to the counter forces it back to 000 in the following cycle.

Design the logic needed to compute the $R G B$ signals given the state $S_{2} S_{1} S_{0}$ of the counter. Use a few gates along with the decoder shown to the right to implement the functions $R, G$, and $B$ as described by the table above.

| color | $R G B$ |
| :---: | :---: |
| RED | 100 |
| YELLOW | 110 |
| GREEN | 010 |
| BLUE | 001 |
| PURPLE | 101 |



Part B (10 points): Draw an abstract transition diagram for a sequence recognizer that identifies the following sequences: 110, 0110, and 1100. In particular, the output $R$ of the sequencer should be 1 whenever the input $B$ has seen any of those three sequences in the last cycles.

Use as few states as possible, explain the meaning of your states, and be sure to specify the starting state.
Note that your diagram states should be labeled with names and output bit, but not with internal state bits (you do not need to pick a representation), but the arcs should be labeled with input combinations.

## Problem 4*** (15 points): Machine Code Analysis

An LC-3 program is located in memory location x3000 to $x 3007$.
The program starts executing at x3000. If we keep track of all values loaded into the MAR as the program executes, we obtain the sequence shown to the right. Such a sequence of values is referred to as a trace.
Fill in the table below with the bits stored in locations x3000 to x3007, then translate the bits to assembly code (fill in the blanks at the bottom of the page).

Some of the bits in the table have been filled in already-use these to deduce the values of the others such that the resulting program leads to the MAR trace shown to the right.

You will need some additional information:

- All registers contain x0000 when the program starts.
- Data stored in location x4FF8 and x5000 are x2012.

MAR trace first value in MAR second value in MAR third value in MAR ...
x3000 x3007 x3001 x3003 x3007 $\times 5000$ x3004 x4FF8

- HALT is TRAP $\times 25$.

| x 3000 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x 3001 |  |  |  |  | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| x 3002 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| x 3003 |  |  |  |  | 0 | 0 | 1 |  |  |  |  |  |  |  |  |  |
| x 3004 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |  |  |  |  |  |  |  |  |  |
| x 3005 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| x 3006 |  |  |  |  | 0 | 0 | 0 |  |  |  |  |  |  |  |  |  |
| x 3007 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Translate the bits to LC-3 assembly code (not RTL) with numeric operands. Do not use labels.
$\qquad$
$\times 3000$
x3001
x3002
x3003
$\times 3004$
x3005
$\times 3006$
$\times 3007$

Problem 5 (25 points): Assembly Code


## Problem 5, continued:

Part B (10 points): The following LC-3 program determines whether or not two strings match (that is, whether or not they have identical contents). The first string starts at memory location $x 4000$, and the second string starts at memory location $x 5000$. Both strings are in the .STRINGZ format. If the two strings are the same, the program terminates with a 1 in R6. If the two strings are different, the program terminates with a 0 in R6. Write one LC-3 assembly instruction into each blank to complete the program. You should not need to define any new labels.


## Problem 5, continued:

Part C (10 points): Write a program in LC-3 assembly language that computes $R E S U L T=|A-4|$. The $|\cdot|$ notation means "absolute value." Your program must have the following characteristics:

- The program must start at memory address x2800.
- The values $A$ and $R E S U L T$ must be placed at the two memory addresses that immediately follow the last instruction in the program. These two addresses must be labeled A and RESULT, respectively.
- The value $A$ must be initialized to 3 .
- The program must produce the correct result for any initial value of $A$ in the range [-1000,1000].
- The program must load the value of $A$, and store the correct $R E S U L T$, from/to the labeled memory locations.
- The program must execute HALT upon completion of this task.
- Appropriately comment your program so that the grader can understand your intent.

Problem 6 (10 points): LC-3 Implementation

Attached to the back of this exam is a copy of the LC-3 state machine (reproduced from the textbook). Tear it off for use with this problem.
Fill in the table below with the appropriate state numbers from that diagram for the ordered sequence of states that are active during the processing of an LC-3 LDR instruction. Note: you may not need all rows of the table below.

Next, for each state, indicate whether each control signal is active (1) or inactive (0). For your convenience, the LC-3 datapath is reproduced below (again from the textbook). DO NOT LEAVE BLANK ENTRIES.

| State \# | Gate.PC | LD.PC | LD.IR | LD.CC | LD.MAR | LD.MDR | GateMDR |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18 |  |  |  |  |  |  |  |
| 33 |  |  |  |  |  |  |  |
| 35 |  |  |  |  |  |  |  |
| 32 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
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Problem 7 (10 points): Critical Paths and Control Unit Design
Part A (3 points): Explain why the critical path through a tree-structured adder such as a Kogge-Stone adder is typically shorter than the critical path through a ripple carry adder.

Part B (4 points): Explain how a memory can be used to implement $N$ Boolean logic functions on $M$ variables. Be specific about the size of memory needed. (You may want to draw a picture.)

Part C (3 points): What is a microinstruction?

This page provided as scratch paper. If you need us to look at this page when grading, indicate this need on the page of the corresponding problem (not here!).

NOTES: RTL corresponds to execution (after fetch!); JSRR not shown

$\mathrm{DR} \leftarrow \mathrm{SR} 1+\mathrm{SR} 2$, Setcc

ADD | 1,1 | 1 | 1 |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 0001 | DR | SR1 | 1 | imm5 |
|  | ADD DR, SR1, imm5 |  |  |  |

DR $\leftarrow$ SR1 + SEXT(imm5), Setcc

AND | 1 | 1 | 1 | 1 | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0101 | DR | SR1 | 0 | 00 | SR2 |
|  | AND DR, SR1, SR2 |  |  |  |  |

DR $\leftarrow$ SR1 AND SR2, Setcc

|  | ${ }^{1} 1{ }^{1}$ |  |  |  | -1 ${ }^{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AND | $\begin{array}{r}0101 \\ \hline 1\end{array}$ | DR | SR1 | 1 | imm5 |

AND DR, SR1, imm5

DR $\leftarrow$ SR1 AND SEXT(imm5), Setcc


BR\{nzp\} PCoffset9
((n AND N) OR (z AND Z) OR (p AND P)):

$$
P C \leftarrow P C+\text { SEXT(PCoffset9) }
$$

JMP | 1100 | 000 | BaseR | 101 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 000000 |  |  |  |
|  | 1 | 1 | 1 | 1 |

$\mathrm{PC} \leftarrow$ BaseR


JSR PCoffset11
$\mathrm{R} 7 \leftarrow \mathrm{PC}, \mathrm{PC} \leftarrow \mathrm{PC}+$ SEXT $($ PCoffset11)


TRAP trapvect8
$\mathrm{R} 7 \leftarrow \mathrm{PC}, \mathrm{PC} \leftarrow \mathrm{M}[$ ZEXT(trapvect8)]

LD

| 101 | 1 | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: | :---: |
| 0010 | DR |  | PCoffset9 |  |

LD DR, PCoffset9
$D R \leftarrow M[P C+S E X T(P C o f f s e t 9)]$, Setcc

LDI

| 1010 | DR |  | 1 | PCoffset9 |
| :---: | :---: | :---: | :---: | :---: |
| 1010 |  |  |  |  |
| 1 | 1 | 1 | 1 | 1 |

LDI DR, PCoffset9
$D R \leftarrow M[M[P C+S E X T($ PCoffset9) $]]$, Setcc

LDR


LDR DR, BaseR, offset6
$\mathrm{DR} \leftarrow \mathrm{M}[$ BaseR + SEXT(offset6)], Setcc

LEA


LEA DR, PCoffset 9

DR $\leftarrow P C+S E X T(P C o f f s e t 9)$, Setcc

NOT

| 101 | DR | SR | 1111 |
| :---: | :---: | :---: | :---: | :---: |
| 1001 | 11111 |  |  |

NOT DR, SR

DR $\leftarrow$ NOT SR, Setcc

ST $\square$ ST SR, PCoffset9

M $[P C+S E X T(P C o f f s e t 9)] \leftarrow S R$

STI


STI SR, PCoffset9

M $[\mathrm{M}[P C+$ SEXT(PCoffset9) $]] \leftarrow$ SR

STR


STR SR, BaseR, offset6

M[BaseR + SEXT(offset6)] $\leftarrow$ SR

## For use with Problem 6.



