ECE199JL Final Exam, Fall 2012

Tuesday 18 December

Name and UIUC Net ID:
SOLUTION

- **Be sure that your exam booklet has 13 pages.**
- **Write your name at the top of each page.**
- **This is a closed book exam.**
- **We have included a scratch sheet and two LC-3 reference pages.**
- **Appendix A of the textbook is available to you on request.**
- **You are allowed FOUR** 8.5×11" **sheets of notes.**
- **Absolutely no interaction between students is allowed.**
- **Show all of your work.**
- **Challenge questions are marked with ***.**
- **Don't panic, and good luck!**

"I think there is a world market for maybe five computers."
—Thomas Watson (Chairman of IBM), 1943

Problem 1	10 points	_____
Problem 2	15 points	_____
Problem 3	15 points	_____
Problem 4	15 points	_____
Problem 5	25 points	_____
Problem 6	10 points	_____
Problem 7	10 points	_____
Total	100 points	_____

Problem 1 (10 points): Representations

Part A (3 points): Explain why an N-bit signed magnitude representation allows you to represent only $2^N - 1$ different numbers.

The number 0 has two representations, sign bit 0 or 1, magnitude 0, so one fewer than 2^N numbers are represented.

Part B (4 points): Two N-bit 2's complement numbers, A and B, are added to find their sum S, as shown to the right.

$$\begin{array}{r} A_{N-1}A_{N-2}\ldots A_2A_1A_0 \\ +\ B_{N-1}B_{N-2}\ldots B_2B_1B_0 \\ \hline S_{N-1}S_{N-2}\ldots S_2S_1S_0 \end{array}$$

Write a Boolean expression for the overflow condition for the addition in terms of the variables shown.

$$A_{N-1}B_{N-1}\overline{S_{N-1}} + \overline{A_{N-1}}\,\overline{B_{N-1}}S_{N-1}$$

Part C (3 points): As you know, addition of two IEEE single-precision floating-point numbers is not associative. In other words, for some values of A, B, and C,

$$(A + B) + C \neq A + (B + C)$$

Give an example of values for A, B, and C for which this lack of associativity holds (*write decimal numbers or scientific notation—you need not translate to the binary representation for IEEE floating-point!*).

A 1

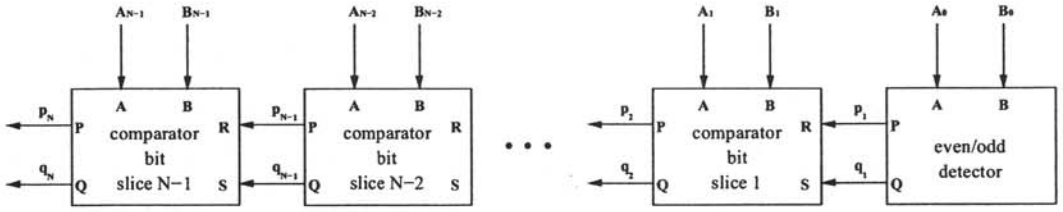
B -1

C 1×10^{-40}

Problem 2 (15 points): Logic

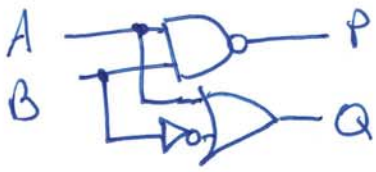
The block diagram below illustrates a specialized N-bit unsigned comparator. The comparator operates on two unsigned numbers, A and B, to produce outputs P_N and Q_N with meanings defined in the table to the right.

P_N	Q_N	Meaning
0	0	$A < B$ and both A and B are odd
0	1	$A \geq B$ and both A and B are odd
1	0	$A < B$ and (A and B are not both odd)
1	1	$A \geq B$ and (A and B are not both odd)



Part A (5 points): Design the even/odd detector. Show all work, including drawing a gate-level diagram implementing outputs P and Q in terms of inputs A and B.

A	B	P	Q
0	0	1	1
0	1	1	0
1	0	1	1
1	1	0	1



could also write eq's as well ...

$P = \overline{AB}$

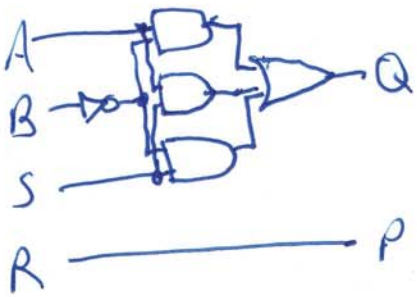
$Q = A + \overline{B}$

Part B (10 points): Design the general bit slice for this comparator. Show all work, including drawing a gate-level diagram implementing outputs P and Q in terms of inputs A, B, R, and S.

Q

RS \ AB	00	01	11	10
00	0	0	0	1
01	1	0	1	1
11	1	0	1	1
10	0	0	0	1

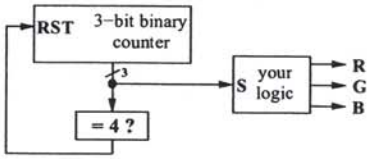
$Q = A\overline{B} + AS + \overline{B}S$



$P = R$

Problem 3 (15 points): Finite State Machines

Part A (5 points): Professor Lumetta promised Professor Cangellaris to design a holiday light display for the new ECE building, but Lumetta has been too busy writing exam problems!

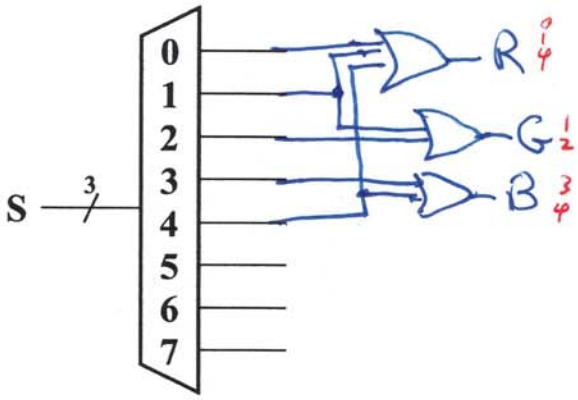


The design to the right shows what he needs: combinational logic that translates the output of a binary counter (counts upward) into RGB signals according to the following repeating sequence in the table below.

The RST input to the counter forces it back to 000 in the following cycle.

Design the logic needed to compute the RGB signals given the state $S_2S_1S_0$ of the counter. Use a few gates along with the decoder shown to the right to implement the functions R, G, and B as described by the table above.

color	RGB
RED	100
YELLOW	110
GREEN	010
BLUE	001
PURPLE	101

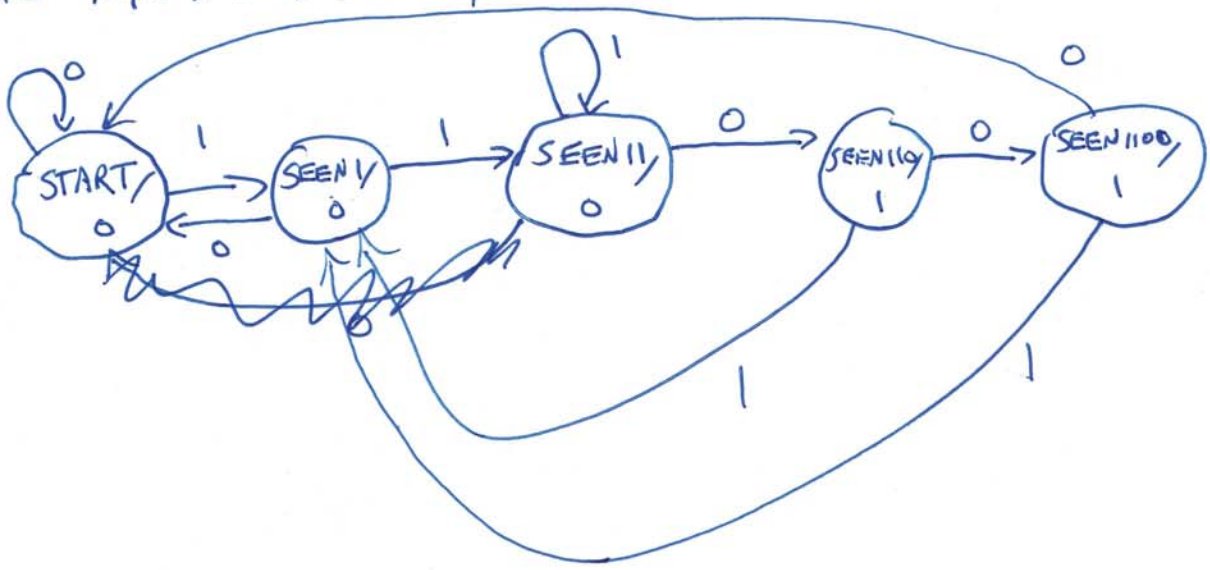


Part B (10 points): Draw an abstract transition diagram for a sequence recognizer that identifies the following sequences: 110, 0110, and 1100. In particular, the output R of the sequencer should be 1 whenever the input B has seen any of those three sequences in the last cycles.

Use as few states as possible, explain the meaning of your states, and be sure to specify the starting state.

Note that your diagram states should be labeled with names and output bit, but not with internal state bits (**you do not need to pick a representation**), but the arcs should be labeled with input combinations.

0110 implies 110, so only need to consider 110 and 1100



Problem 4* (15 points): Machine Code Analysis

An LC-3 program is located in memory location x3000 to x3007.

The program starts executing at x3000. If we keep track of all values loaded into the MAR as the program executes, we obtain the sequence shown to the right. Such a sequence of values is referred to as a trace.

MAR trace	
first value in MAR	x3000
second value in MAR	x3007
third value in MAR	x3001
...	x3003
	x3007
	x5000
	x3004
	x4FF8
	x3005
	x3006
	x3002

Fill in the table below with the bits stored in locations x3000 to x3007, then translate the bits to assembly code (fill in the blanks at the bottom of the page).

Some of the bits in the table have been filled in already—use these to deduce the values of the others such that the resulting program leads to the MAR trace shown to the right.

You will need some additional information:

- All registers contain x0000 when the program starts.
- Data stored in location x4FF8 and x5000 are x2012.
- HALT is TRAP x25.

x3000	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0
x3001	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1
x3002	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1
x3003	1	0	1	either 0	0	0	1	0	0	0	0	0	0	0	1	1
x3004	0	1	1	0	0	1	0	0	0	0	1	1	1	0	0	0
x3005	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1
x3006	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0
x3007	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0

Translate the bits to LC-3 assembly code (not RTL) with numeric operands. Do not use labels.

x3000 LD R0, #6

x3001 JSR #1

x3002 HALT

x3003 LDI R1, #3 / STI R1, #3

x3004 LDR R2, R0, #-8

x3005 ADD R2, R2, ~~#1~~ R1

x3006 JMP R7 / RET

x3007 .FILL x5000 (or AND R0, R0, R0)

Problem 5 (25 points): Assembly Code

Part A (5 points): The LC-3 assembler produces the symbol table below when assembling the code shown to the right. Fill in the blank entries.

```
// Symbol table
// Scope level 0:
// Symbol Name        Address
// ----------------   -----------
//
// MAIN               x3000
// LD1                x3001
// ST1                x3006
// LD2                x3007
// ST2                x300C
// INIT               x300F
//
// FOO                x3015
// CALC               x301B
// NEXT               x3020
// DONE               x3024
// RETN               x302A
// OP1                x302B
// OP2                x302C
// EXT                x302D
// FLAG               x302E
//
// NAME               x302F
//
// SIGN               x3039
//
// MASK               x303A
```

```
                .ORIG    x3000

MAIN    AND     R3,R3,#0
LD1     LD      R1,OP1
        LD      R4,SIGN
        AND     R5,R1,R4
        BRz     ST1
        JSR     FOO
ST1     ADD     R2,R1,#0
LD2     LD      R1,OP2
        LD      R4,SIGN
        AND     R5,R1,R4
        BRz     ST2
        JSR     FOO
ST2     ADD     R1,R1,#0
        AND     R3,R3,#1
        ST      R3,FLAG
INIT    AND     R4,R4,#0
        AND     R5,R5,#0
        ADD     R5,R5,#1
        AND     R6,R6,#0
        JSR     CALC
        HALT

FOO     ADD     R3,R3,#1
        LD      R4,EXT
        ADD     R1,R1,R4
        NOT     R1,R1
        ADD     R1,R1,#1
        RET
CALC    ADD     R3,R4,#-6
        BRzp    DONE
        AND     R3,R5,R2
        BRz     NEXT
        ADD     R6,R1,R6
NEXT    ADD     R5,R5,R5
        ADD     R1,R1,R1
        ADD     R4,R4,#1
        BRnzp   CALC
DONE    LD      R3,FLAG
        BRz     RETN
        NOT     R6,R6
        ADD     R6,R6,#1
        LD      R4,MASK
        AND     R6,R6,R4
RETN    RET

OP1     .FILL   x000B
OP2     .FILL   x0007
EXT     .FILL   xFF00
FLAG    .BLKW   #1
NAME    .STRINGZ "Read this"
SIGN    .FILL   x0080
MASK    .FILL   x4FFF

        .END
```

Problem 5, continued:

Part B (10 points): The following LC-3 program determines whether or not two strings match (that is, whether or not they have identical contents). The first string starts at memory location x4000, and the second string starts at memory location x5000. Both strings are in the .STRINGZ format. If the two strings are the same, the program terminates with a 1 in R6. If the two strings are different, the program terminates with a 0 in R6. Write one LC-3 assembly instruction into each blank to complete the program. *You should not need to define any new labels.*

```
            .ORIG   x3000
            LD      R1, STRING1
            JSR     LENGTH
            ADD R4, R0, #0      ; part A   (or R3)
            LD      R1, STRING2
            JSR     LENGTH
            ADD R3, R0, #0      ; part B   (or R4)
            NOT     R4, R4
            ADD     R4, R4, #1
            ADD     R4, R4, R3
            BRnp    NO

            LD      R1, STRING1
            LD      R2, STRING2
CONTINUE    LDR     R3, R1, #0
            LDR     R4, R2, #0
            BRz YES             ; part C
            NOT     R4, R4
            ADD     R4, R4, #1
            ADD     R4, R4, R3
            BRnp    NO
            ADD R1, R1, #1      ; part D   (can swap)
            ADD R2, R2, #1      ; part E
            BRnzp CONTINUE

YES         AND R6, R6, #0
            ADD R6, R6, #1
            BRnzp DONE

NO          AND R6, R6, #0
DONE        HALT

            ; a subroutine
LENGTH      AND     R0, R0, #0
COUNT       LDR     R5, R1, #0
            BRz     RETURN
            ADD     R0, R0, #1
            ADD     R1, R1, #1
            BRnzp   COUNT
RETURN      RET

STRING1     .FILL x4000
STRING2     .FILL x5000
            .END
```

(R3 or R4 are in each)

(any matching value is ok)

Problem 5, continued:

Part C (10 points): Write a program in LC-3 assembly language that computes $RESULT = |A - 4|$. The $|\cdot|$ notation means "absolute value." Your program must have the following characteristics:

- The program must start at memory address x2800.
- The values A and $RESULT$ must be placed at the two memory addresses that immediately follow the last instruction in the program. These two addresses must be labeled A and RESULT, respectively.
- The value A must be initialized to 3.
- The program must produce the correct result for any initial value of A in the range [-1000,1000].
- The program must load the value of A, and store the correct $RESULT$, from/to the labeled memory locations.
- The program must execute HALT upon completion of this task.
- Appropriately comment your program so that the grader can understand your intent.

```
.ORIG x2800

LD  R0, A
ADD R0, R0, #-4      ; calculate A-4
BRzp DONE            ; if it's ≥ 0, store it...
NOT R0,R0            ; negate negative values (4-A)
ADD R0, R0, #1

DONE  ST R0, RESULT  ; store result

      HALT           ; done

A     .FILL #3
RESULT .BLKW #1
      .END
```

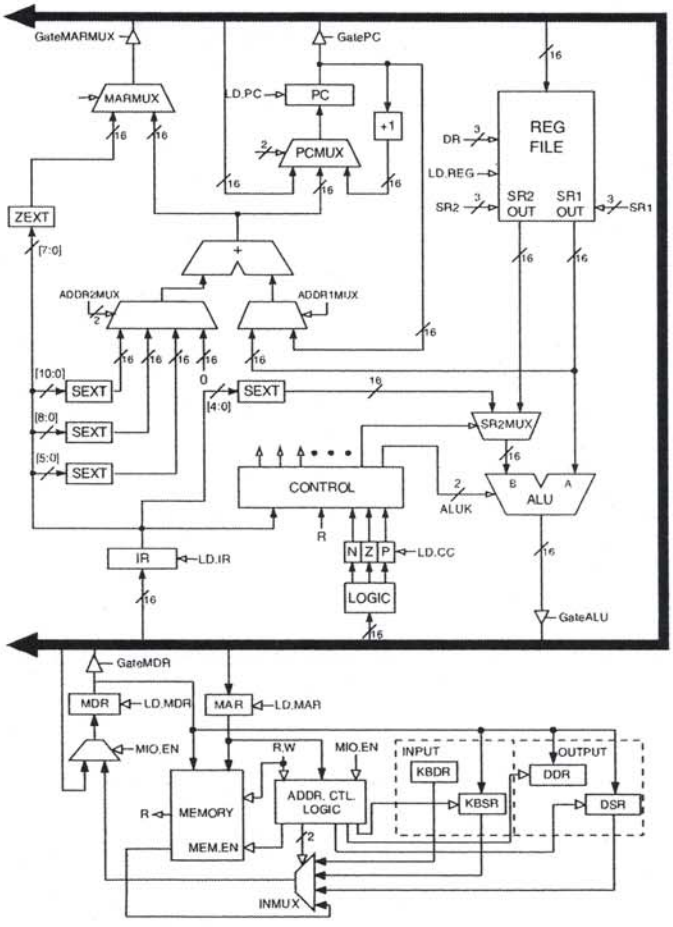
Problem 6 (10 points): LC-3 Implementation

Attached to the back of this exam is a copy of the LC-3 state machine (reproduced from the textbook). *Tear it off for use with this problem.*

Fill in the table below with the appropriate state numbers from that diagram for the ordered sequence of states that are active during the processing of an LC-3 LDR instruction. **Note: you may not need all rows of the table below.**

Next, for each state, indicate whether each control signal is active (1) or inactive (0). For your convenience, the LC-3 datapath is reproduced below (again from the textbook). **DO NOT LEAVE BLANK ENTRIES.**

State #	Gate.PC	LD.PC	LD.IR	LD.CC	LD.MAR	LD.MDR	GateMDR
18	1	1	0	0	1	0	0
33	0	0	0	0	0	1	0
35	0	0	1	0	0	0	1
32	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0
25	0	0	0	0	0	1	0
27	0	0	0	1	0	0	1

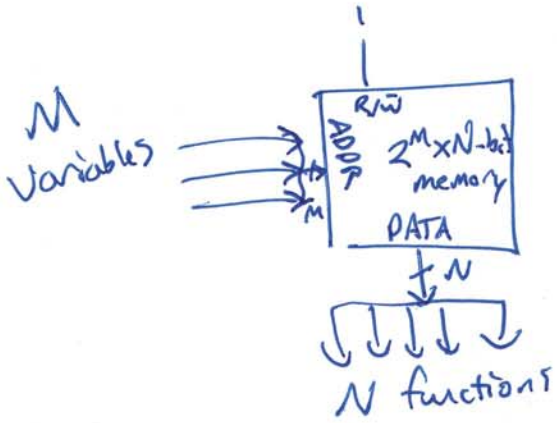


Problem 7 (10 points): Critical Paths and Control Unit Design

Part A (3 points): Explain why the critical path through a tree-structured adder such as a Kogge-Stone adder is typically shorter than the critical path through a ripple carry adder.

Tree structure balances delay among all paths by breaking repeatedly into halves (or any size, really), so proportional to $\log_2$(# of bits).
Ripple carry delay along carry chain is proportional to # of bits.

Part B (4 points): Explain how a memory can be used to implement N Boolean logic functions on M variables. Be specific about the size of memory needed. (*You may want to draw a picture.*)



Part C (3 points): What is a microinstruction?

A set of control signals that implement the RTL for a particular state in a processor's state machine.

This page provided as scratch paper. If you need us to look at this page when grading, indicate this need **on the page of the corresponding problem** (not here!).

NOTES: RTL corresponds to execution (after fetch!); JSRR not shown

ADD

ADD DR, SR1, SR2

DR ← SR1 + SR2, Setcc

ADD

ADD DR, SR1, *imm5*

DR ← SR1 + SEXT(imm5), Setcc

AND

AND DR, SR1, SR2

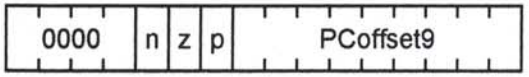
DR ← SR1 AND SR2, Setcc

AND

AND DR, SR1, *imm5*

DR ← SR1 AND SEXT(imm5), Setcc

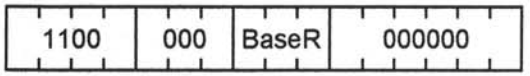
BR



BR{nzp} *PCoffset9*

((n AND N) OR (z AND Z) OR (p AND P)):
PC ← PC + SEXT(PCoffset9)

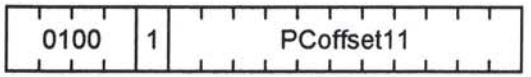
JMP



JMP BaseR

PC ← BaseR

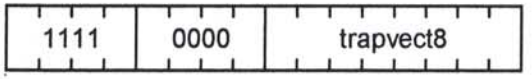
JSR



JSR *PCoffset11*

R7 ← PC, PC ← PC + SEXT(PCoffset11)

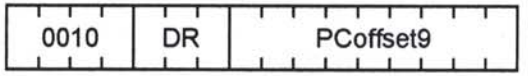
TRAP



TRAP *trapvect8*

R7 ← PC, PC ← M[ZEXT(trapvect8)]

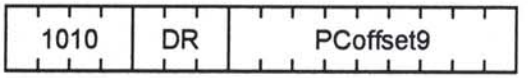
LD



LD DR, *PCoffset9*

DR ← M[PC + SEXT(PCoffset9)], Setcc

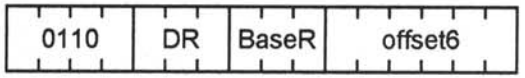
LDI



LDI DR, *PCoffset9*

DR ← M[M[PC + SEXT(PCoffset9)]], Setcc

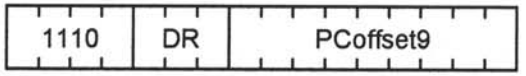
LDR



LDR DR, BaseR, *offset6*

DR ← M[BaseR + SEXT(offset6)], Setcc

LEA



LEA DR, *PCoffset9*

DR ← PC + SEXT(PCoffset9), Setcc

NOT

NOT DR, SR

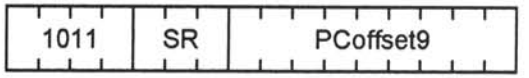
DR ← NOT SR, Setcc

ST

ST SR, *PCoffset9*

M[PC + SEXT(PCoffset9)] ← SR

STI



STI SR, *PCoffset9*

M[M[PC + SEXT(PCoffset9)]] ← SR

STR

STR SR, BaseR, *offset6*

M[BaseR + SEXT(offset6)] ← SR

For use with Problem 6.