MATLAB

Polynomials - mostly

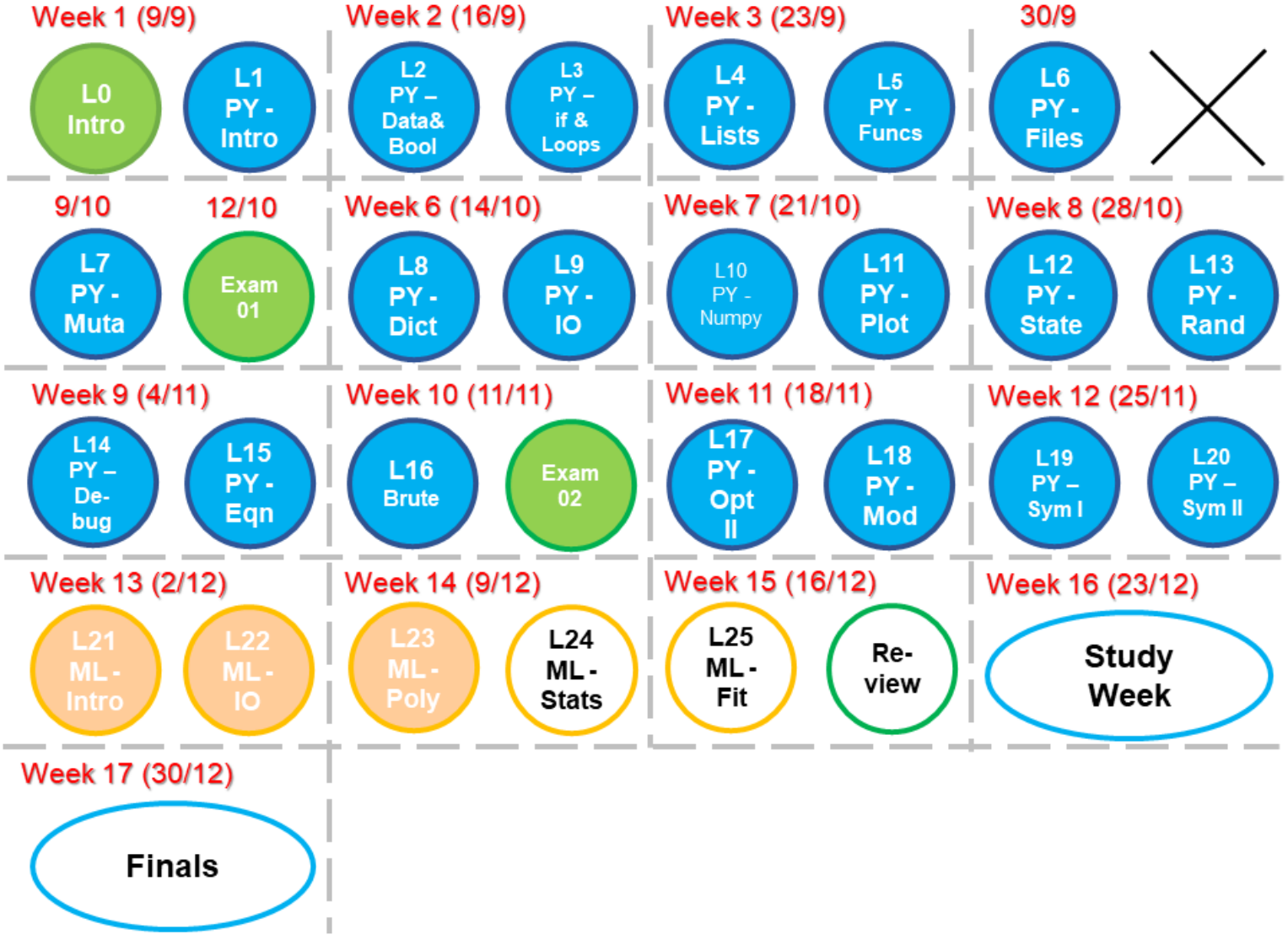
Announcements

quiz: `quiz23` due on Tues 10/12

lab: `lab` on Fri 13/12

hw: `hw12` due Wed 11/12

Roadmap


Week 1 (9/9)
L0 Intro
L1 PY - Intro
Week 2 (16/9)
L2 PY – Data& Bool
L3 PY – if & Loops
Week 3 (23/9)
L4 PY - Lists
L5 PY - Funcs
30/9
L6 PY - Files
9/10
L7 PY - Muta
12/10
Exam 01
Week 6 (14/10)
L8 PY - Dict
L9 PY - IO
Week 7 (21/10)
L10 PY - Numpy
L11 PY - Plot
Week 8 (28/10)
L12 PY - State
L13 PY - Rand
Week 9 (4/11)
L14 PY – De-bug
L15 PY - Eqn
Week 10 (11/11)
L16 Brute
Exam 02
Week 11 (18/11)
L17 PY - Opt II
L18 PY - Mod
Week 12 (25/11)
L19 PY – Sym I
L20 PY – Sym II
Week 13 (2/12)
L21 ML - Intro
L22 ML - IO
Week 14 (9/12)
L23 ML - Poly
L24 ML - Stats
Week 15 (16/12)
L25 ML - Fit
Re-view
Week 16 (23/12)
Study Week
Week 17 (30/12)
Finals

Objectives

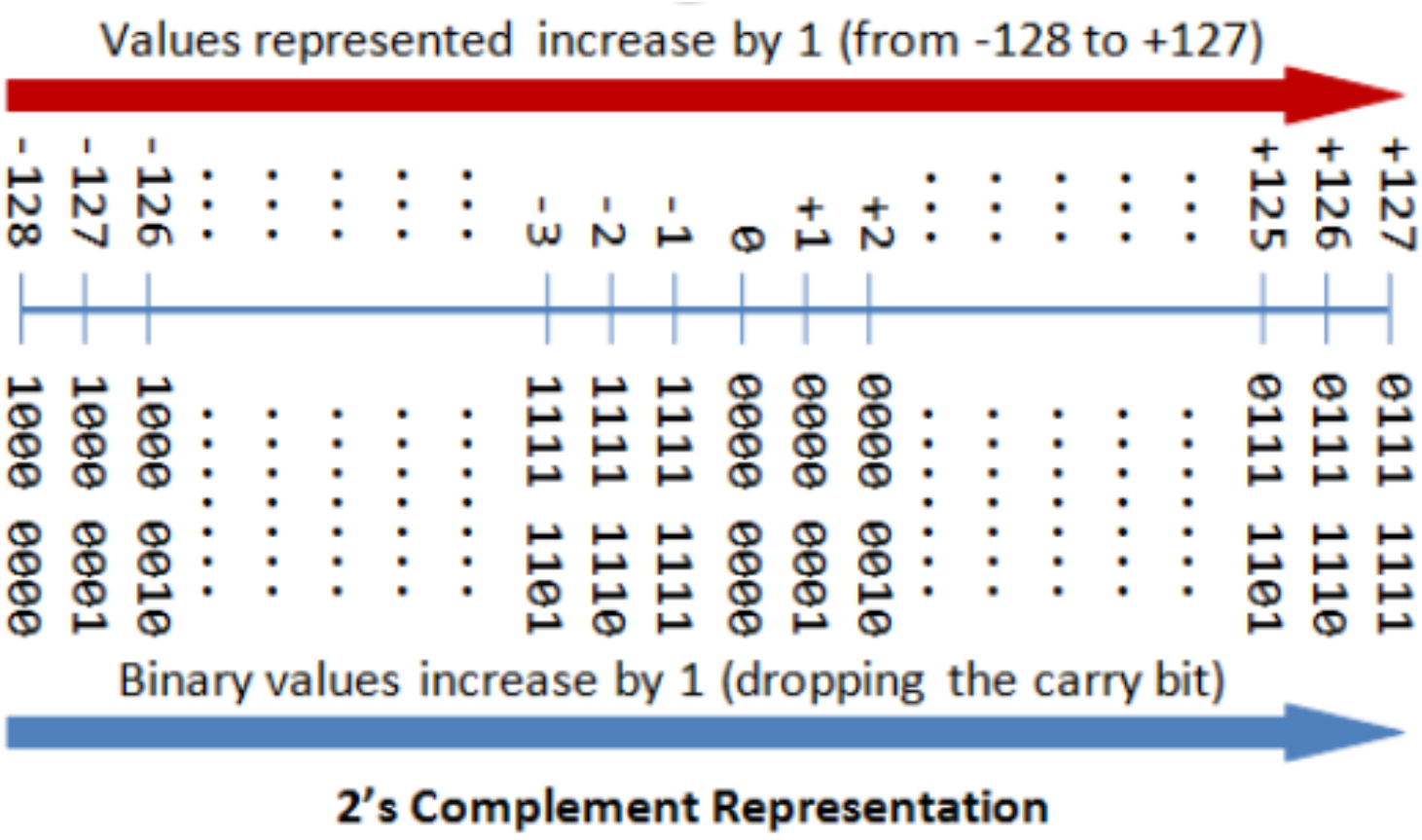
A. Create and evaluate polynomial arrays.
B. Find function zeros and polynomial roots.
C. Minimize functions within bounds (`fminbnd`).
D. Numerically differentiate data using `polyder`.
E. Numerically integrate data using `polyint`.

MATLAB Review

Some stuff

1. Function - write in `XXX.m` file; only 1 function in 1 file; preferably `name`.m is the same `name` of the function
2. How to change directory in matlab
3. recursive vs loop

int8 (optional)



Question 0

```
D = [ 1 2 ; 3 4 ];
g = D .^ 2;
```

What is the value of `g`?

A `[ 1 2; 3 4 ]`

B `[ 1 4; 3 4 ]`

C `[ 1 4; 9 4 ]`

D `[ 1 4; 9 16 ]`

E None of the above (an error occurs)

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D `[ 1 4; 9 16 ]` ★★★

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x = [ 1 0 ]';
f = D * x;
```

What is the value of `f`?

A `[ 1 ; 3 ]`

B `[ 1 0; 3 0]`

C `[ 1 ; 2 ]`

D `[ 1 2 ]`

E `[ 1 2 ; 0 0 ]`

F None of the above (an error occurs)

Question 1

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D = [ 1 2 ; 3 4 ];
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What is the value of `f`?

A `[ 1 ; 3 ]` ⋆⋆⋆

B `[ 1 0; 3 0]`

C `[ 1 ; 2 ]`

D `[ 1 2 ]`

E `[ 1 2 ; 0 0 ]`

F None of the above (an error occurs)

Question 2

```
D = [ 1 2 ; 3 4 ];
x = [ 1 0 ]';
g = D .* x;
```

What is the value of `g`?

A `[ 1 ; 3 ]`

B `[ 1 0; 3 0]`

C `[ 1 ; 2 ]`

D `[ 1 2 ]`

E `[ 1 2 ; 0 0 ]`

F None of the above (an error occurs)

Question 2

```
D = [ 1 2 ; 3 4 ];
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g = D .* x;
```

What is the value of `g`?

A `[ 1 ; 3 ]`

B `[ 1 0; 3 0]`

C `[ 1 ; 2 ]`

D `[ 1 2 ]`

E `[ 1 2 ; 0 0 ]` ⋆⋆⋆why?

F None of the above (an error occurs)

Question 3

```
D = [ 1 2 ; 3 4 ];
x = [ 1 0 ];
g = D .* x;
```

What is the value of `g`?

A `[ 1 ; 3 ]`

B `[ 1 0; 3 0]`

C `[ 1 ; 2 ]`

D `[ 1 2 ]`

E `[ 1 2 ; 0 0 ]`

F None of the above (an error occurs)

Question 3

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D = [ 1 2 ; 3 4 ];
x = [ 1 0 ];
g = D .* x;
```

What is the value of `g`?

A `[ 1 ; 3 ]`

B `[ 1 0; 3 0]` ⋆⋆⋆why?

C `[ 1 ; 2 ]`

D `[ 1 2 ]`

E `[ 1 2 ; 0 0 ]`

F None of the above (an error occurs)

Question 4

```
D = [ 1 2 ; 3 4 ];
x = [ 1 0 0 ];
g = D .* x;
```

What is the value of `g`?

A `[ 1 ; 3 ]`

B `[ 1 0; 3 0]`

C `[ 1 ; 2 ]`

D `[ 1 2 ]`

E `[ 1 2 ; 0 0 ]`

F None of the above (an error occurs)

Question 4

```
D = [ 1 2 ; 3 4 ];
x = [ 1 0 0 ];
g = D .* x;
```

What is the value of `g`?

A `[ 1 ; 3 ]`

B `[ 1 0; 3 0]`

C `[ 1 ; 2 ]`

D `[ 1 2 ]`

E `[ 1 2 ; 0 0 ]`

F None of the above (an error occurs) ⋆⋆⋆why?

Question 5

```
D = [ 1 2 ]';
x = [ 1 0 0 ];
g = D .* x;
```

What is the value of `g`?

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```
D = [ 1 2 ]';
x = [ 1 0 0 ];
g = D .* x;
```

What is the value of `g`?

```
g =

     1     0     0
     2     0     0
```

Question 6

```
D = [ 1 2 ; 3 4; 5 6; 7 8 ];
x = [ 1 2; 3 4 ];
g = D .* x;
```

What is the value of `g`?

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```
D = [ 1 2 ; 3 4; 5 6; 7 8 ];
x = [ 1 2; 3 4 ];
g = D .* x;
```

What is the value of `g`?

```
Error
```

elementwise operations

A Can operate when the two matrices are of the same dimensions

B Also when the other is a scalar (== number)

C sometimes when the other is a 1D row vector or 1D column vector

Polynomials

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How do you solve this?

$$x^2 - 4$$

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How about this?

$$3x^3 + x^2 - 4$$

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```
[ 3 1 0 -4 ]
```

Polynomials

$$a_1 x^n + a_2 x^{n-1} + \cdots + a_{n-1} x^2 + a_n x + a_{n+1}$$

A polynomial is a length $(n+1)$ array.

Question

$$5x^5 + 5x^4 + 4x^3 + 2x$$

How would this polynomial be represented in MATLAB?

A `[ 5 5 4 0 2 ]`

B `[ 2 0 4 5 5 ]`

C `[ 5 5 4 0 2 0 ]`

D `[ 0 2 0 4 5 5 ]`

Question

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How would this polynomial be represented in MATLAB?

A `[ 5 5 4 0 2 ]`

B `[ 2 0 4 5 5 ]`

C `[ 5 5 4 0 2 0 ]` ⋆

D `[ 0 2 0 4 5 5 ]`

Polynomials

$$3x^3 + x^2 - 4$$

To evaluate a polynomial for a particular value of x, use `polyval`:

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To evaluate a polynomial for a particular value of x, use `polyval`:

```
polyval( [ 3 1 0 4 ],5 )  % for x = 5

ans =

   404
```

Polynomials

To plot a polynomial:

```
p = [ -1 0 4 0 ]    %coefficients of polynomial

x = -5:0.1:5        %x-value
y = polyval( p,x )  %y-value

plot( x,y,'r--' )
```

Polynomials - multiply

$$u = 3x^2 - 1$$

$$v = 2x + 5$$

$$w = u * v = (x^2 - 1)(x + 5) = 6x^3 + 15x^2 - 2x - 5$$

```
u = [ 3 0 -1 ];
v = [ 2 5 ];
w = u*v ???
```

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```
u = [ 3 0 -1 ];
v = [ 2 5 ];
w = u*v ???
```

To multiply a polynomial, `*` won't work; use `conv` instead:

```
w = conv( u,v )
```

Polynomials - integrate

To integrate a polynomial between [a, b], use `polyint`:

$$x^3 - x$$

between [0, 1]

```
integrand = [ 1 0 -1 0 ];
```

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$$\int_0^1 dx\, x^3 - x$$

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antiderivative = polyint( integrand );
```

Polynomials - integrate

To integrate a polynomial between [a, b], use `polyint`:

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$$\int_0^1 dx\, x^3 - x = \frac{x^4}{4} - \frac{x^2}{2}\Big|_0^1 = \frac{1}{4} - \frac{1}{2} - \frac{0}{4} + \frac{0}{2} = -\frac{1}{4}$$

```
integrand = [ 1 0 -1 0 ];
antiderivative = polyint( integrand );
integral_l = polyval( antiderivative,1 );
integral_r = polyval( antiderivative,0 );
integral = integral_l - integral_r;

integral =
   -0.2500
```

Polynomials - dy/dx

To differentiate a polynomial, use `polyder`:

$$p(x) = x^5 - x^4 + x^3 - x^2 + x - 1$$

$$\frac{dp}{dx} = 5x^4 - 4x^3 + 3x^2 - 2x + 1$$

```
polynomial = [ 1 -1 1 -1 1 -1 ];
derivative = polyder( polynomial );
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polynomial = [ 1 -1 1 -1 1 -1 ];
derivative = polyder( polynomial );

derivative =

     5    -4     3    -2     1
```

Optimize/Solving

Optimization

Given a function $f(\underline{x})$, find $\underline{x} = \underline{x}^*$ such that $f(\underline{x}^*)$ is maximized (or minimized, or equal to zero).

The goal is to search the domain for the $\underline{x}^*$ which yields the solution $f(\underline{x}^*)$.

Find Roots

$$3x^3 + x^2 - 4$$

To obtain the roots of the polynomial (as you would use when factoring it), use `roots`:

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```
roots( [ 3 1 0 4 ] )
```

Find Polynomial from roots

$$p(x) = x^5 - x^4 + x^3 - x^2 + x - 1$$

$$p(x) = (x-1)\left(x^2 - x + 1\right)\left(x^2 + x - 1\right)$$

```
polynomial = [ 1 -1 1 -1 1 -1 ];
r = roots( polynomial )
```

Find Polynomial from roots

$$p(x) = x^5 - x^4 + x^3 - x^2 + x - 1$$

$$p(x) = (x-1)\left(x^2 - x + 1\right)\left(x^2 + x - 1\right)$$

```
polynomial = [ 1 -1 1 -1 1 -1 ];
r = roots( polynomial )
```

`poly` goes the other way: a polynomial array from a set of roots.

`poly(r)` to get the above equation back

Find Solutions

$$\cos x = e^{-x} - 4$$

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Rewrite this equation as a function in standard form:

$$f(x) = \cos x - e^{-x} + 4$$

Finding Solutions

$$f(x) = \cos x - e^{-x} + 4$$

or in MATLAB

```
f = (@x) cos(x) - exp(-x) + 4
```

1. Solve the equation with

   `fzero` (if seeking zeros)

   `fminbnd` (if seeking minima).

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1. Solve the equation with
   `fzero` (if seeking zeros)
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2. `x = fzero( f,x0 )` tries to find a zero of fun near `x0`.

How do we find all the roots in a range??? Think about it.

Finding Solutions

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1. Solve the equation with
   `fzero` (if seeking zeros)
   `fminbnd` (if seeking minima).
2. `x = fzero( f,x0 )` tries to find a zero of fun near `x0`.

   How do we find all the roots in a range??? Think about it.
3. `x = fminbnd( f,x1,x2 )` returns a value `x` that is a local minimium of the function (defined in `f`) in the interval `[x1,x2]`.

NOTE: `fminbnd( f,x1,x2 )` with `f` not `@f`

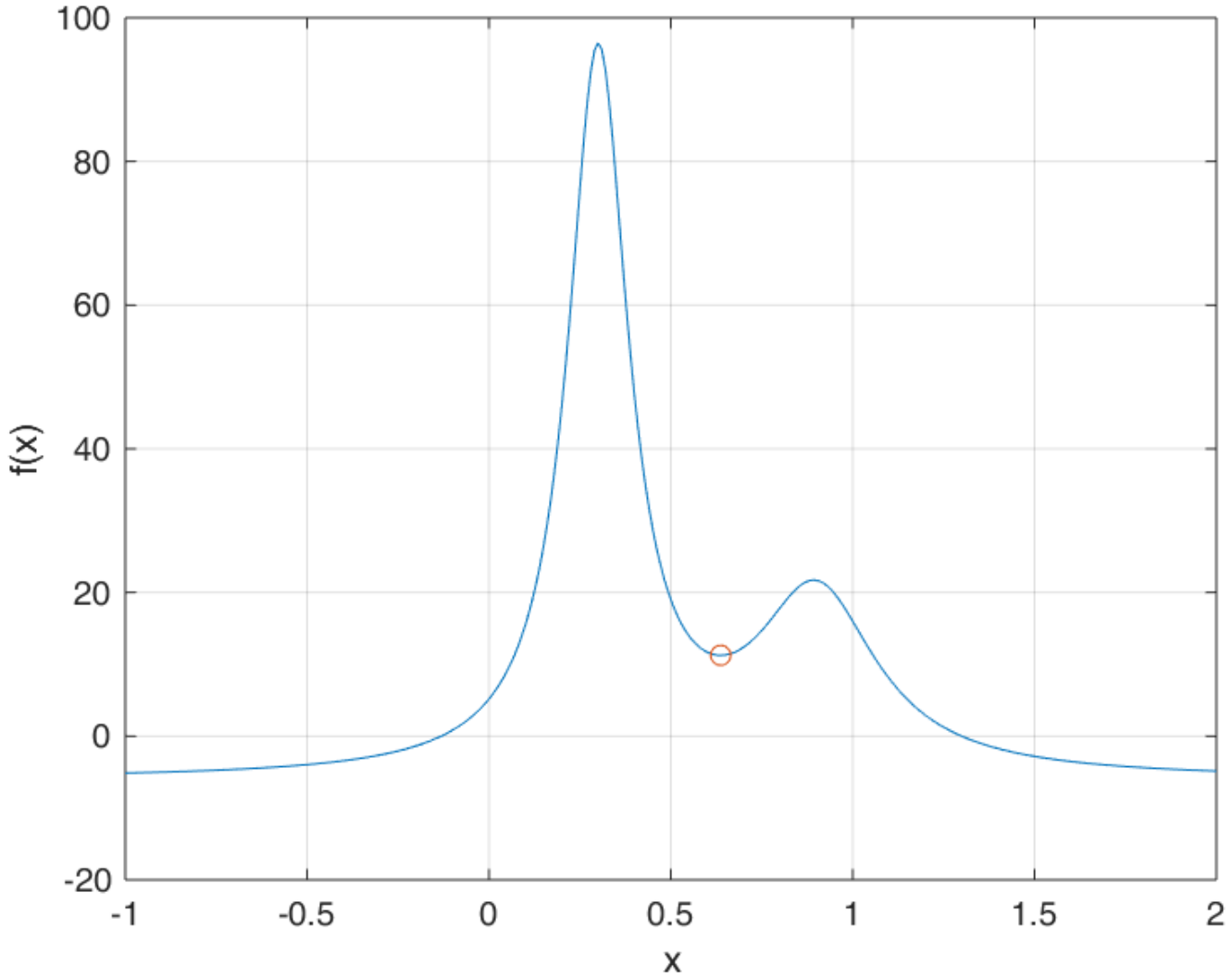
Optimization

```
x = -1:.01:2;
y = humps( x );    % a matlab function

figure
plot( x,y )
xlabel( 'x' )
ylabel( 'f(x)' )
grid on

xstar = fminbnd( @humps,0.3,1 )
[ xstar,ystar ] = fminbnd( @humps,0.3,1 )
```

Optimization



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xstar = fminbnd( @humps,0.3,1 )
[ xstar,ystar ] = fminbnd( @humps,0.3,1 )
```

Note the `@` inside `fminbnd( @humps,0.3,1 )`

How do we maximize a function??? Think about it

The @ sign

Compare:
M1:
y = humps(x); % a matlab or your function in .m file
xstar = fminbnd(`@humps`, 0.3, 1)

and
M2:
f = (@x) cos(x) - exp(-x) + 4
xstar = fminbnd(`f`, 0.3, 1)

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and
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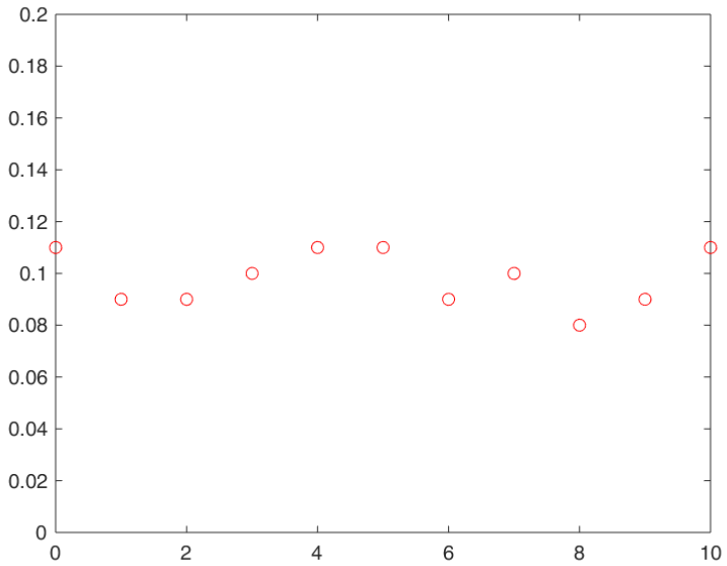
xstar = fminbnd(`f`, 0.3, 1)

Use `@` when your function is not defined in the same code.

Numerical Integration

Numerical Integration

```
% simulated data set of measurements
x = linspace( 0,10,11 );
y = [ 0.11 0.09 0.09 0.10 0.11 0.11
          0.09 0.10 0.08 0.09 0.11 ];
figure;
plot( x,y,'ro' );
ylim( [ 0 0.2 ] );
```



Numerical Integration

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% simulated data set of measurements
x = linspace( 0,10,11 );
y = [ 0.11 0.09 0.09 0.10 0.11 0.11
          0.09 0.10 0.08 0.09 0.11 ];

figure;
plot( x,y,'ro' );
ylim( [ 0 0.2 ] );
```

To integrate a data set, use `trapz`:

```
integral = trapz( x,y );
```

Summary

A. Express polynomial as arrays, `polyval`, `conv`
B. Find `roots`, create `poly`nomial
C. Differentiate using `polyder`, integrate with `polyint`
D. Find zeros using `fzero`
E. Minimize functions within bounds (`fminbnd`).
F. Numerical integrate using `trapz`.